CE 6405 Soil Mechanics Notes

UNIT-I

INTRODUCTION

Nature of soil – Problems with soil – phase relation – sieve analysis –sedimentation – analysis – Atterberg limits – classification for engineering purposes – BIS classification system – soil compaction – factors affecting compaction – field compaction methods

TWO MARK QUESTIONS AND ANSWERS

1. **Define: Water Content (w)**

Water content is defined as the ratio of weight of water to the weight of solids in a given mass of soil.

2. Density of Soil: Define

Density of soil is defined as the mass the soil per unit volume.

3. Bulk Density: Define (ρ)

Bulk density is the total mass M of the soil per unit of its total volume.

4. Dry Density: Define (ρ_d)

The dry density is mass of soils per unit of total volume of the soil mass.

5. Define: Saturated Density (ρ_{sat})

When the soil mass is saturated, is bulk density is called saturated density

6. Define: Submerged Density (ρ')

The submerged density is the submerged mass of the soil solids per unit of total volume of the soil mass.

7. Define: Unit Weight of Soil Mass

The unit mass weight of a soil mass is defined as it s weight per unit volume.

8. Bulk Unit Weight: Define (γ)

The bulk weight is the total weight of a soil mass per unit of its total volume.

9. Dry Unit Weight: Define (γ_d)

The dry unit weight ifs ht weight of solids per unit of its total volume of the soil mass.

10. Unit Weight of Solids: Define (γ_s)

The unit weight of soil solids is the weight of soil solids per unit volume of solids.

11. What Is Submerged Unit Weight (γ')

The submerged unit weight is the submerged weight of soil solids per unit of the total volume of soils.

12. What Is Saturated Unit Weight (γ_{sat})

Saturated unit weight is the ratio of the total weight of a saturated soil sample to its total sample.

13. What Is Void Ratio? (e)

Void ratio of a given soil sample is the ratio of the volume of soil solids in the given soil mass.

14. What is Porosity? (n)

The porosity of a given soil sample is the ratio of the volume of voids to the total volume of the given soil mass.

15. Degree of saturation: Define (Sr)

The degree of saturation is defined as the ratio of the volume of water present in a given soil mass to the total volume of voids on it.

16. Define: percentage of air voids (n_a)

Percentage of air voids is defined as the ratio of the volume of air voids to the total volume of soil mass.

17. Air content: Define (a_c)

The air content is defined as the ratio of volume of air void to the volume of voids.

18 .Define: Density Index (I_D) or Relative Compactive

The density index is defined as the ratio of the differences between the voids ratio of the soil in the loosest state and its natural voids ratio ratio & to the differences between voids ratio in the loosest and densest states.

19. What is compaction?

Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer strata of contact by mechanical means in order to decrease the porosity (or voids ratio) of the soil and thus increase its dry density.

20. Aim of the compaction

i) To increase the shear strength soil
ii) To improve stability and bearing capacity

iii) To reduce the compressibility
iv) To reduce the permeability of the soil.

21. What are the methods available for sieve analysis?

a) Dry sieve Analysis
b) Wet sieve analysis

22. Atterberg limits: define

The limit at which the soil, changes from one state to another state, is termed as atterberg limits.

23. Liquid limit: define

Is the water content at which the soil, changes from liquid to plastic state liquid.

24. What is plastic limit?

The maximum water content at which, soil changes from plastic to semi-solid state.

25. Define: percentage of air voids (n_a)

Percentage of air voids is defined as the ratio of the volume of air voids to the total volume of soil mass.

16 MARKS QUESTIONS AND ANSWERS

1. Writes notes on nature of soil?

a) The stress strain relation ship for a soil deposit is nonlinear .hence the difficulty in using easily determinable parameter to describe its behavior.

b) Soil deposits have a memory for stress they have undergone in their geological history. Their behavior is vastly influenced by their stress history; time and environment are other factors which may alter their behavior.

c) Soil deposit being for from homogenous , exbit properties which vary from location

d) As soil layers are buried and hidden from view. One has to rely on test carried out on small samples that can be taken; there is no grantee that the soil parameters are truly representative of the field strata

e) No sample is truly undisturbed. in a soil which is sensitive to disturbance; the behavior submersed from the laboratory tests may not reflect the likely behavior of the field stratum.

2. Explain the problems related to soils.

Soils is, the ultimate foundation material which supports the structure the proper functioning of the structure will, therefore, depend critically element resting on the subsoil. Here the term foundation is used in the conventional sense. A substructure that distributes the load to the ultimate foundation, namely, the soil.

From ancient times, man has used soil for the construction of tombs, monuments, dwellings and barrages for storing water. In the design and construction of underground structures such as tunnels, conduits, power houses, bracings for excavations and earth retaining structure, the role of soil is again very crucial, since the soil is in direct contact with the structure, it acts as a medium of load transfer and hence for any analysis of forces acting on such structure, one has to consider the aspects of stress distribution through the soil.

The structure, two causes stresses and strain in the soils, while the stability of the structure itself is affected by soil behavior. The class problems where the structure and soil mutually interact are known as soil- structure interaction problems. There are a host of other civil engineering problems related to soils. For designing foundations for machines such as turbine, compressors, forges etc…. which transmit vibrations to the foundation soil, one has to understand the behavior of soil under vibratory loads.

The effect of quarry blasts, earthquakes and nuclear explosions on structures is greatly influenced by the soil medium through which the shock waves traverse. In these parts of the world which experience freezing temperatures, problems arise because the soil expand upon freezing and exert a force on the structure in the contact with them.

Thawing (due to melting ice) of the soil results in a soil results in a loss of strength in the soil. Structure resting on these soils will perform satisfactorily only if measures are taken to prevent frost heave or designed to withstand the effects of the freezing and thawing

3. Prove that: $\mathbf{e} = \dfrac{wG}{S_r}$

Soil element in terms of e_w and e

From figure: e_w -- volume of water

e -- Volume of voids

G - specific gravity

The volume of solids is equal to unity

$$S_r = \frac{V_w}{V_v} = \frac{e_w}{e}$$

$$e_w = e\,S_r \quad \rightarrow (1)$$

$e_w = e$, when fully saturated sample.

$$w = W_w/W_d = \frac{e_w \gamma_w}{\gamma_s . 1}$$

$$G = \frac{\gamma_s}{\gamma_w} \rightarrow \gamma = G\gamma_w$$

$$w = \frac{e_w \gamma_w}{G\gamma_w} = \frac{e_w}{G}$$

$$e_w = w.G \rightarrow (2)$$

From equation (1) & (2)

$$e = \frac{wG}{S_r} \rightarrow (3)$$

When fully saturated sample, Sr = 1 and $w = w_{sat}$

$$e = w_{sat.} G$$

4. Prove that :

$$n_a = \frac{e(1-S_r)}{1+e}$$

$$n_a = \frac{V_a}{V}$$

$$V_a = Vv - V_w = e - e_w$$

$$V = Vs + Vv = 1+e$$

$$n_a = \frac{e-e_w}{1+e}$$

$e_w = e\ Sr$ from equ $\rightarrow (1)$

$n_a = e - e\,S_r / (1+e) = e\,(1 - S_r) / (1+e)$

$n_a = e\,(1 - S_r) / (1+e) \rightarrow (4)$

5. Prove that:

$n_a = n\,a_c$

$a_c = \dfrac{v_a}{v_v}$;;; $n = \dfrac{v_v}{v}$

$n_a = \dfrac{v_a}{v_v} = n\,a_c \rightarrow (5)$

6. Prove that:

$\gamma_d = \dfrac{G\gamma_w}{1+e}$

$\gamma_d = \dfrac{wd_v}{V}$

$\gamma_d = \dfrac{\gamma_s v_s}{V}$

Vs = 1(refer soil element in terms of ‘e’ figure)

V = (1+e)

$\gamma_d = \dfrac{\gamma_s . 1}{1+e}$

$\gamma_s = G.\,\gamma_\omega$

$\gamma_d = \dfrac{G\gamma_w}{1+e} \rightarrow (6)$

Note:

$$1+e = \frac{G\gamma_w}{1+e} -$$

$$e = \frac{G\gamma_w}{1+e} - 1$$

From figure (ii) soil element in terms of 'n'

$$Vs = \frac{G\gamma_w(1-n)}{1} = (1-n)\,G\,\gamma_\omega \rightarrow \quad (7)$$

7 .a. Prove that:

$$\gamma_{sat} = G\,\gamma_\omega(1-n) + \gamma_\omega.n$$

$$\gamma_{sat} = w_{sat} / V$$

$$= (W_d + W w) / V$$

$$= \frac{\gamma_s V_{s.} + \gamma_w V_w}{V}$$

From, fig (ii)

Vs = 1, Vw = e, and V = 1+e

$$\gamma_{sat} = \frac{\gamma_s.1 + \gamma_w e}{1+e}$$

$$= \frac{G\gamma_w + \gamma_w.e}{1+e}$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w.}{1+e} \rightarrow \quad (8)$$

From fig (ii)

Vs=1-n, V w= n, V=1

$$\gamma_{sat} = \frac{\gamma_s (1-n) + \gamma_{w.}.n}{1} \rightarrow (9)$$

7. b. Prove that:

$$\gamma = \frac{(G + e\ S_r)\gamma_w}{1+e}$$

$$\gamma = W/V = \frac{\gamma_s.V_s + \gamma_w V_w}{V}$$

Refer: figure (iii)

Vs = 1, Vw = e, and V = 1+e

$$\gamma = \frac{\gamma_s.1 + \gamma_w e}{1+e}$$

$\gamma_s = G\ \gamma_w$ & e_w = e Sr

$$\gamma = \frac{G.\gamma_w + \gamma_w eSr}{1+e} \rightarrow (10)$$

if the soil is perfectly dry,Sr = 0

When Sr =1

γ Become $\gamma_{sat} = \dfrac{(G+e)\gamma_w}{1+e} \rightarrow (11)$

8. a. Prove that: $\gamma' = \dfrac{(G-1)\gamma_w}{1+e}$

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$= \frac{(G+e)\gamma_w}{1+e} - \gamma_w$$

$$= \frac{(G-1)\gamma_w}{1+e} \rightarrow (12)$$

8. b.Prove that : $\gamma_d = \dfrac{\gamma}{1+w}$

Water content : w = Ww / Wd

1+ w = $\dfrac{Ww+Wd}{Wd}$ = W/ Wd

Wd= W / (1+w) $\rightarrow$ (13)

γ_d =Wd /V = W / (1+w) V

$\gamma_d = \dfrac{\gamma}{1+w} \rightarrow$ (!4)

9. Prove that

$\gamma' = \gamma_d$ - (1-n) γ_w

From fig (iii)

$(Wd)_{sub}$=1. γ_s - 1. γ_w

$(Wd)_{sub}$=(G-1) γ_w

V= 1+e

From equation (12)

γ' = $(Wd)_{sub.}$ /V

$$= \frac{(G-1)\gamma_w}{1+e} = \frac{G\gamma_w}{1+e} - \frac{\gamma_w}{1+e}$$

From equation (6)

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

1 / (1+e) =1-n

$$\gamma' = \gamma_d - (1\text{-}n)\ \gamma_w \rightarrow (15)$$

10.a . Prove that

$$\gamma = \gamma_d + Sr(\gamma_{sat} - \gamma_d)$$

From equation (10)

$$\gamma = \frac{G.\gamma_w + \gamma_w eSr}{1+e}$$

$$= \frac{G.\gamma_w}{1+e} + \frac{\gamma_w eSr}{1+e}$$

$$= \gamma_d + Sr\ \left[\frac{(G+e)\gamma_w}{1+e} - \frac{G.\gamma_w}{1+e}\right]$$

$$= \gamma_d + Sr\ ((\gamma_{sat} - \gamma_d) \rightarrow (16)$$

10.b. Prove that:

$$\gamma_d = \frac{G.\gamma_w}{1+w_{sat}.G}$$

From equation (6)

$$\gamma_d = \frac{G.\gamma_w}{1+e}$$

From equation (3)

$$\gamma_d = \frac{G\ \gamma_w}{1+\frac{w}{Sr}} \rightarrow (17)$$

When Sr= 1, $\gamma_d = \dfrac{G\,\gamma_w}{1+W_{sat.}G} \rightarrow$(17.a)

11. Prove that

$$\gamma_d = \frac{(1-n)G\,\gamma_w}{1+w.G}$$

From fig (i)

V= Va + Vw + Vs

= Va +Ww / γ_w + Wd / γ_s

$$1 = \frac{Va}{V} + \frac{w.Wd}{\gamma_{w.}V} + \frac{Wd}{\gamma_s.V}$$

$$1 = \frac{Va}{V} + \frac{w.\gamma_d}{\gamma_{w.}} + \frac{\gamma_d}{\gamma_s}$$

$$1- \frac{Va}{V} = \frac{w.\gamma_d}{\gamma_{w.}} + \frac{\gamma_d}{G\gamma_w}$$

$$1 \quad n_a = \frac{\gamma_d}{\gamma_{w.}}(w+\frac{1}{G})$$

$$\gamma_d = \frac{(1-n_a)G.\gamma_w}{(1+wG)}$$

12. A soil sample has a porosity of 40% .the specific gravity of solids 2.70, Calculate (a) void ratio

(b) Dry density

(c) Unit weight if the soil is 50% saturated

(d) Unit weight if the soil is completely saturated

Solution:

(a) e $= \dfrac{n}{1-n} = \dfrac{0.4}{1-0.4} = 0.667$

(b) $\gamma_d = \frac{G.\gamma_w}{1+e} = \frac{2.7*9.81}{1+0.667} = 15.89$ KN / m^3

(c) e = $\frac{w.G}{Sr}$

w = $\frac{eSr}{G} = \frac{.667*0.5}{2.7} = 0.124$

$\gamma = \gamma_{d.}(1+w) = 15.89*1.124 = 17.85$ KN / m^3

(d) When the soil is fully saturated

e= $w_{sat.}$G

w_{sat} = e / G = 0.667 /2.7 = 0.247

γ_{sat} = G γ_w(1-n) + γ_w.n

=2.7 * 9.81(1- 0.4) + 9.81*0.4 = 19.81 KN / m^3

13. An undisturbed sample of soil has a volume of 100 cm^3 and mass of 190.g. On oven drying for 24 hrs, the mass is reduced to 160 g. If the specific gravity grain is 2.68, determine the water content, voids ratio and degree of saturation of the soil.

Solution:

Mw= 190-160= 30 g

Md = 160 g

W= Mw / Md= 30 / 160 = 0.188 = 18.8 %

Mass of moist soil = M = 190 g

Bulk density = M/ V = 190 / 100= 1.9 g/ cm^3

γ = 9.81 * ρ = 9.81* 1.9= 18.64 KN / m^3

$\gamma_d = \frac{\gamma}{1+w}$

$$= \frac{18.64}{1+.188} = 15.69 \text{ KN / m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1$$

$$= \frac{2.68 * 9.81}{15.69} - 1 = 0.67$$

$$Sr = \frac{w.G}{e} = \frac{0.188 * 2.68}{0.67} = .744 = 74.45\%$$

14. The in-situ density of an embankment, compacted at a water content of 12 % was determined with the help of core cutter. The empty mass of the cutter was 1286 g and the cutter full of soil had a mass of 3195 g, the volume of the cutter being 1000 cm^3. Determine the bulk density, dry density and the degree of saturation of the embankment.

If the embankment becomes fully saturated during rains, what would be its water content and saturated unit weight / assume no volume change in soil on saturation .Take the specific gravity of the soil as 2.70.

Solution:

Mass of soil in cutter

M= 3195- 1286= 1909 g

Bulk density ρ = M/ V = 1909 / 1000 = 1.909 g/ cm^3

Bulk unit weight γ = 9.81* ρ

= 9.81* 1.909= 18.73 KN / m^3

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.73}{1+0.12} = 16.72 \ \textbf{KN / m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{16.72} - 1 = .0.584$$

$$Sr = \frac{w.G}{e} = \frac{0.12 * 2.7}{0.584} = .555 = 55.5\%$$

At saturation:

Since the volume remains the same, the voids ratio also remains unchanged.

$e = w_{sat.} G$

$w_{sat.} = e / G = 0.584 / 2.7 = 0.216 = 21.6\%$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.7+.584)9.81}{1+.584} = 20.34 \text{ KN/ m}^3$$

15. The in-situ percentage voids a sand deposit is 34 percent .for determining the density index , dried sand from the stratum was first filled loosely in a 1000 cm^3 mould and was then vibrated to give a maximum density . The loose dry mass in the mould was m1610 g and dense dry mass at maximum compaction was found to be 1980 g. Determine the density index if the specific gravity of the sand particles 2.67

Solution,

n = 34%

e= n / (1-n) = 0.34 /(1-0.34) = 0.515

$$(\gamma_d)_{max} = \frac{1980}{1000} * 9.81 = 19.42 \text{ KN/ m}^3$$

$$(\gamma_d)_{min} = \frac{1610}{1000} * 9.81 = 15.79 \text{ KN/ m}^3$$

$$e_{min} = \frac{G\gamma_w}{(\gamma_d)_{min}} - 1 = \frac{2.67 * 9.81}{19.42} - 1 = 0.349$$

$$e_{max} = \frac{G\gamma_w}{(\gamma_d)_{max}} - 1 = \frac{2.67 * 9.81}{15.79} - 1 = 0.659$$

$I_D = (e_{max} - e) / (e_{max} - e_{min}) = (0.659 - 0.515) / (0.659 - 0.349)$

$= 0.465 = 46.5\ \%$

aaaaa

16. The mass specific gravity (apparent gravity) of a soil equals 1.64. The specific gravity of solids is 2.70. Determine the voids ratio under assumption that the soil is perfectly dry .What would be the voids ratio, if the sample is assumed to have a water content of 8 percent?

Solution:

When the sample is dry

$$\text{Gm} = \frac{\gamma_d}{\gamma_w} = 1.64$$

$$\gamma_d = 1.64 * \gamma_w = 1.64 * 9.81 = 16.09 \text{ KN/ m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{16.09} - 1 = 0.646$$

When the sample has water content

w = 8 %

$$\gamma = 1.64 * \gamma_w = 1.64 * 9.81 = 16.09 \text{ KN/ m}^3$$

$$\gamma = \frac{\gamma}{1+w} = \frac{16.09}{1+0.08} = 14.9 \text{ KN/ m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{14.9} - 1 = 0.78$$

17. A natural soil deposit has a bilk unit weight of 18.44 KN/ m^3 , water content of 5 % .calculate the amount of water required to the added to 1 m^3 of soil to raise the water content to 15 %. Assume the void ratio to remain content .What will then be the degree of saturation? Assume G= 2.67

Solution:

γ = 18.44 KN/ m^3; w = 5%

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.44}{1+0.05} = 17.56 \text{ KN/ m}^3$$

w = Ww / Wd = 0.05

For one cubic meter of soil, v = 1 m^3

Wd = γ_d . V= 17.56 * 1 = 17.56 KN.

Ww= 0.05* Wd= 0.05 * 17.56= 0.878 KN

Vw = Ww / γ_d =0.878 / 9.81 = 0.0895 m^3

Later, when w = 15 %

Ww = w. Wd = 0.15 * 17.56 = 2.634 KN

Vw = Ww / γ_w = 2.534 / 9.81 = 0.2685 m^3

Hence additional water required to raise the water content from 5 % to15% = 0.2685 – 0.0895 = 0.179 m= 179 liters.

$$\text{Voids ratio, } e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.67 * 9.81}{17.56} - 1 = 0.49$$

After the water has been added 'e 'remains the same

Sr = w.G / e = 0.15 * 2.67 / 0.49= 0.817= 81.7%

18. Calculate the unit weighs and specific gravities of solids of (a) soil composed of pure quartz and (b) a soil composed of 60 % quartz, 25% mica, and 15% iron oxide. Assume that both soils are saturated and have voids 0f 0.63. Take average and for iron oxide = 3.8

Solution

a) For the soil composed of pure Quartz,

G for quartz = 2.66

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.66+0.63) \; * 9.81}{1+0.63} = 19.8 \text{ KN/ m}^3$$

b) for the composite soil,

G average= (2.66*0.6) + (3.* 0.25) + (3.8 *0.15)
=1.6 + 0.75 + 0.57= 2.92

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.92+0.63) \; * 9.81}{1+0.63} = 21.36 \text{ KN/ m}^3$$

19. A soil has a bulk unit weight of 20.22 KN/ m^3 and water content of 15%. Calculate the water content if the soil partially dries to a unit weight of 19.42 KN/ m^3. and voids ratio remains unchanged.

Solution:

Before drying,

γ = 20.11 KN/ m^3

γ_d = 20.11 / (1 + 0.15) = 17.49 KN/ m^3

Since after drying, e does not change, V and $\gamma_{d,}$ are the same,

$$\gamma = \gamma_d(1+ w)$$

$$1+w = \gamma / \gamma_d = 19.42 / 17.49 = 1.11$$

$$w = 1.11 – 1 = 11\%$$

20. A cube of dried clay having sides 4 cm long has a mass of 110 g. The same cubes of soil, when saturated at unchanged volume, has mass of 135 g. Draw the soil element showing the volumes and weights of the constituents, and then determine the specific gravity of soil solids and voids ratio.

Solution:

Volume of soil = $(4)^3$ = 64 m^3
Mass water after saturation
= 135- 110= 25 g

Volume of solids = 25 cm^3
Volume of solids = 64- 25 = 39 m^3
Ms = 110 g

$$G = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w} = \frac{Ms}{Vs*1} = \frac{110}{39*1} = 2.82$$

$$e = \frac{Vs}{Vd} = 25/39 = 0.64$$

21. a. Explain Dry sieve analysis

The soil should be oven-dry, it shouldn't contain any lump, if necessary, it should be pulverized. If organic matters in the soil;, it taken air – dry instead of oven dry. The sample is sieved through a 4.75 mm IS sieve .the portion retained on the sieve is gravel fraction or plus 4.75 mm material .then gravel fraction is sieved through the set of coarse sieves manually or mechanical shaker.

The minus 4.75mm fraction is sieves through the set if fine sieves .the sample is placed in the top sieves and the set of sieves is kept on a mechanical shaker. Normally, 10 minutes of shacking is sufficient for most soils. The mass of soil retained on each sieve and on pan is obtained to the nearest 0.1gm

Suitability: cohesion less soils with little or no fines.

21. b. Explain wet sieve Analysis.

If the soil contains a saturated a substantial quantity of fine particles,
A wet analysis required. A soil sample in the required quantity is taken, using a rifer an
dried in an oven . The dried sample is taken in a tray and sacked with water. The samples stirred

and lift soaking period of at least one hour.

The slurry is taken sieved through a 4.75 mm IS sieve, and washed with a jet of water . The material retained on the sieve is the gravel faction. The material retained on the 75 μ sieve is collected and dried an oven. It is then sieved through the set of the fine sieves of the size 2 mm,1 mm, 600 μ ,425 μ , 212 μ , 150 μ ,and 75 μ

The material that would have been retained on the pan is equal to the total mass of soil minus the sum of the masses of material retain on all sieves. .

22. Explain the analysis of sedimentation by pipette method.

The method is based on stokes law.

Stokes law:

The velocity with which a grain settles down in suspension , all other factors being equal , is dependent upon the shape, weight and size of the grains.

Assumption:

The coarser particles, will settle more than the finer ones.

There are 3 forces are there.

i) Drag force
ii) Weight of the sphere
iii) Buoyant force

The resisting force due to drag resistant offered by a fluid.

$$R = 6 \, \Pi \, \eta \, r \, u$$

Where,

η = dynamic viscosity in $KN.s/m^2$

r= radius in m

u = velocity in m/s

Weight of the sphere = $\frac{4}{3}\pi r^3 \gamma_s$

= $\frac{4}{3}\pi r^3 \rho_s g$

Buoyant force = $\frac{4}{3}\pi r^3 \gamma_{\omega s}$

= $\frac{4}{3}\pi r^3 \rho_{\omega s} g$

Equilibrium of forces in vertical direction

W = U + FD

$$\frac{4}{3}\pi r^3 \rho_s g = \frac{4}{3}\pi r^3 \rho_{\omega s} g + 6\ \pi\ \eta\ \text{r u}$$

$$V = \frac{2}{9}\frac{r^2}{\eta}(\rho_s - \rho_w)g$$

$$= \frac{1}{18}\frac{D^2}{\eta}(\rho_s - \rho_w)g$$

$$D = \sqrt{\frac{18\eta v}{(\rho_{s-}\rho_w)g}}$$

$$= \sqrt{\frac{18\eta v}{\gamma_s - \gamma_w}}$$

Now

$$D = \sqrt{\frac{18\eta v}{\rho_\omega g(G-1)}}$$

$$= \sqrt{\frac{18\eta v}{\gamma_\omega (Ss-1)}}$$

If spherical particle falls through a height He

V = He/ 60 t

$$\text{He/ 60 t} = \frac{1}{18}\frac{D^2}{\eta}(\rho_s - \rho_w)g$$

$$= \frac{1}{18}\frac{D^2}{\eta}(G - 1)\rho_\omega$$

$$D = \sqrt{\frac{0.3\eta He}{g(G-1)\rho_{\omega} t}}$$

$$D = M\sqrt{\frac{He}{t}}$$

Where M is a factor, equal to $\sqrt{\frac{0.3\eta}{g(G-1)\rho_{\omega}}}$

23. What are the limitations of sedimentation analysis?

i) The sedimentation analysis gives the particle size in terms of equilant diameter, which is less than the particle size given by sieve analysis. The soil particles are not spherical.

ii) Stokes' law is applicable only when the liquid is infinite. The presence of walls of the jar affects the results to same extent.

iii) In stokes law, it has been assumed that only one sphere settles, and there is no interference from other spheres. In sedimentation analysis, as many particles settled simultaneously, there is some interference.

iv) The sedimentation analysis cannot be used for particles larger than 0.2 mm as turbulent conditions develop and stokes law isn't applicable.

24. Explain the soil classification

a) Classification based on grain size
b) Textural classification
c) AASHTO classification
d) Unidefined soil classification

a) Classification based on grain size

This classification based on grain size. In this system the terms clay, silt, and gravel are used to indicate only particle size and not to signify nature of soil type.

		sand					
		Very fine	Fine	medium	coarse		

0.005 0.05 0.1 0 .25 0.5 1.0 2.0

b) Textural classification

The classification of soil exclusively based on particle size and their percentage distribution is known as textural classification .this system specifically names the soil depending on the percentage of sand, silt and clay.

b) AASHTO classification

This system is developed based on the particle size and plasticity characteristic of soil mass .A soil is classified by proceeding from left to right on the chart to find first the group into which the soil test data will fill. Soil having fine fraction are further classified based on their group index

Group index = (F-35) [0.2 + 0.005(L.L – 40)] + 0.01(F-15) (P.I-10)

F- Percentage passing 0.075 mm size

LL-Liquid Limit

P.I – Plasticity Limit

d) Unified soil classification system

This system is based on the both grain size and plasticity characteristic of soil. IS system divides soils into three major groups coarse grained and organic soils and other miscellaneous soil materials. Coarse grained soils are those with more than 50 % of the material larger than 0.075 mm size .Coarse grained soils are further divided grained soils are further divided in to gravels , sands. Fine grained soils are those for which more than 50 % of soil finer than 0.05mm sieve size.

They divided into three subdivisions as silt, clay and organic salts and clays. based on the their plasticity nature they are added with L,M,N and H symbol to indicate low plastic , medium plastic and high plastic respectively.

25. Explain the BIS classification for soil system

Indian standard classification (ISC) system adopted by Bureau of Indian Standards is in many aspects.

Soils are divided onto three broad divisions

(i) Coarse-grained soils, when 50% or more of the total material by weight is retained on 75 µ IS Sieve

(ii) Fine – grained soils, when more then 50% of the total material passes 75µ IS sieve

(iii) If the soil is highly organic and contains a large percentage of organic matter and particles of decomposed vegetation, it is kept in a separate category marked as peat.

1. Coarse – grained soils.
   Coarse – grained soils are subdivided into gravel and sand. The soil is termed

gravel and sand. The soil is termed gravel (G) where more than 50% Coarse fraction (plus 75 μ) is retained on 4.75mm IS sieve ,and termed sand (s) if more than 50 % of the coarse friction is smaller than 4.75 mm IS sieve.

2. Fine Grained
   Fine – grained soils are further divided into three subdivisions, depending upon the values of the liquid limit.
   a) Silts and clays of low compressibility – liquid limit less than 35 (Represented by symbol H)
   b) Silts and clays of medium compressibility- these soils have liquid limit greater than 35 but less than 50.
   c) Silts and clays of high compressibility- these soils have liquid limit greater than 50(Represented by symbol H).

26. Different between consolidation and compaction

S.NO	CONSOLIDATION	COMPACTION
1	It is a gradual process of reduction of volume under sustained, static loading.	It is a rapid of reduction of volume mechanical mean such as rolling , tamping , vibration.
2	It causes a reduction in volume of a saturated soil due to squeezing out of water from the soil.	In compaction, the volume of partially saturated soil decreases of air the voids at the unaltered water content.
3	Is a process which in nature when saturated soil deposits are subjected to static loads caused by the weight of the building	Is an artificial process which is done to increase the density of the soil to improve its properties before it is put to any use.

27. What are the factors affecting compaction? Explain in brief?

i) Water content

a) At lower water content, the soil is stiff and others more resistance to compaction.

b) As water content is increases, the soil particles get lubricated.

c) Dry density of the soil increases with increases in the water content till the optimum water content is reached.

d) After the optimum water content is reached, it becomes more difficult to force air out and to further reduce the air voids.

ii) Amount of compaction

At water content less than the optimum, the effect of increased compaction is more predominant. At water content more than optimum, the volume of air voids becomes almost constant and the effect of increased compaction is of significant.

iii) Type of soil

In general, coarse – grained soils can be compacted to higher dry density than fine grained soils. With the addition of even a small quantity of fines to a coarse-grained soil, the soil attains a much higher dry density for the same compactive effort.

Cohesive soils have air voids .Heavy clays of very high plasticity have very low dry density and very high optimum water content

iv) Method of compaction

The dry density achieved depends not only upon the amount of compactive effort; the dry density will depend upon whether the method of compaction utilizes kneading action, dynamic or static action.

v)Admixture

The compaction characteristic of the soils is improved by adding other materials known as admixtures. Ex; lime, cement and bitumen

.

28. What are the different methods of compaction adopted in the field?

i) Tampers.

A hand operated tamper consists of block iron, about 3 to 5 Kg o mass, attached to a wooden rod. The tamper is lifted for about 0.30m and dropped on the soil to be compressed. Mechanical Tampers operated by compressed air or gasoline power.

ii) Rollers

a) smooth – wheel rollers
b) pneumatic – tyred rollers
c) Sheep- foot rollers.

a) smooth – wheel rollers
Smooth – wheel rollers are useful finishing operations after compaction of fillers and for compacting granular base causes of highways.

b) Pnumatic – tyred rollers
Pneumatic – tyred rollers use compressed air to develop the required inflation pressure.
The roller compactive the soil primarily by kneading action. These rollesrs are effecting for compacting cohesive as well as cohesion less soils.

c) Sheep – foot rollers
The sheep – foot roller consists of a hollow drum with a large number of small projections (known as feet) on its surface. The drums are mounted on a steel frame. The drum can fill with water or ballast increases the mass. The contact pressure is generally between 700 to 4200 KN/m^2.

(FOR IV – SEMESTER)

UNIT – II

SOIL WATER AND WATER FLOW

Soil water – various forms – Influence of clay minerals – Capillary rise-suction-Effective stress concepts in soil-total, neutral and effective stress distribution in soil –permeability - Darcy's law- Permeability measurement in the laboratory – quick sand condition – seepage – Laplace Equation – Introduction to flow nets – properties and uses – Application to simple problems.

Two Marks Questions and Answers

1. Define soil water.

Water present in the voids of a soil mass is called soil water.

2. State the types of soil water.

i. Free water (or) Gravitational water

ii. Held water

   a. Structural water
   b. Absorbed water
   c. Capillary water.

3. Define free water and held water.

Free water:

Water that is free to move through a soil mass under the influence of gravity is known as free water.

Held water:

Held water is the part of water held in soil pores by some forces existing within the pores: such water therefore is not free to move under gravitational forces.

4. Define structural, Adsorbed and capillary water.

Structural water:

Structural water is the water chemically combined in the crystal structure of the soil mineral and can be removed only by breaking the structure.

Adsorbed water:

Adsorbed water, also termed as the hygroscopic water (or) the contact moisture (or) surface bound moisture. It is the part which the soil particles freely adsorb from atmosphere by the physical forces of attraction and is hold by the force of adhesion.

Capillary water:

Water held in the interstices of soil due to capillary forces is called capillary water.

5. Draw the diagrammatic representation of water molecules.

The soil particles carry a net negative charge. Due to this charge, they attract water. The water in the soil system that is not under significant forces of attraction from the soil particle is pore water.

6. Define capillary action (or) capillarity:

It is the phenomenon of movement of water in the interstices of a soil due to capillary forces. The capillary forces depend upon various factors depend upon various factors such as surface tension of water, pressure in water in relation to atmospheric pressure and thee size and conformation of soil pores.

7. Define contact moisture.

Water can also be held by surface tension round the point of contact of two particles (spheres) capillary water in this form is known as contact moisture (or) contact capillary water.

8. Compute the maximum capillary tension for a tube 0.05 mm in diameter.

Solution:

The maximum capillary height at 4° C is given by

$$(hc)_{\max} = \frac{0.3084}{d} = \frac{0.3084}{0.005} = 61.7cm = 0.617m$$

$\therefore$ Capillary tension = $(hc)_{max}\ \gamma_{\omega}$ = 0.617 x 9.81

= 6.05 x KN/m^3

9. Compute the height of capillary rise in a soil whose D_{10} is 0.1 mm and voids ratio is 0.60.

Solution:

Let the average size of the void be d mm.

Volume of each sphere of solids maybe assumed proportional to D_{10}^3. Since the voids ratio is 0.6, the volume of rods space, corresponding to the unit of volume of solids, will be proportional to 0.60 D_{10}^3 . But volume of each void space is also proportional to d^3.

Hence $d^3 = 0.60\ D_{10}^3$

$$d = (0.60)^{1/3} D_{10}$$

$= 0.845 \times D_{10}$

$= 0.845 \times 0.1$

d = 0.0845 mm = 0.00845 cm

$$hc = \frac{0.3084}{d} cm$$ at 4° C.

$$hc = \frac{0.3084}{0.00845} = 36.5cm$$

10. When water at 20º is added to a fine sand and to a silt, a difference in capillary rise of 25 cm is observed between the two soils. If the capillary rise in fine sand is 25 cm, calculate the difference in the size of the voids of the two soils.

Solution:

Using suffix 1 for sand 2 for silt,

$hc_1 = 25$ cm

$hc_2 = 25+25 = 50$ cm

$$d_1 = \frac{0.2975}{hc_1} = \frac{0.2975}{25} = 0.0119\ cm$$

$$d_2 = \frac{0.2975}{hc_2} = \frac{0.2975}{50} = 0.00595\ cm$$

$\therefore$ Difference in the size of the voids

$d_1 - d_2 = 0.0119 - 0.00595$

$= 0.00595$ cm

11. The capillary rise in soil A with D_{10} = 0.06 mm is 60 cm. Estimate the Capillary rise in soil B with D_{10} = 0.1 mm, assuming the same voids ratio in both the soils.

Solution:

Let the size of voids be d.

Now $V_S \propto D_{10}^{\ 3}$

$$V_v = eV_s$$

$$e = \left(\frac{d}{D_{10}}\right)^3 \qquad \text{(i)}$$

For soil A,

$$d = \frac{0.3084}{h_c} = \frac{0.3084}{60} = 5.14 \times 10^{-3} cm$$

= 5.14 x 10^{-2} mm

Substitute it in (i), we get,

$$e = \left(\frac{d}{D_{10}}\right) = \left(\frac{5.14 \times 10^{-2}}{0.06}\right)^3 = 0.629$$

Now for soil B $d = (e)^{1/3} D_{10}$

$$= (0.629)^{1/3} \times 0.1$$

$$= 0.857 \times 0.1 = 0.0857 mm$$

= 0.00857 cm

12. Define Permeability.

Permeability is defined as the property of a porous material which permits the passage of water (or) other fluids through its interconnecting voids.

A material having continuous voids is called permeable. Grovels are highly permeable while stiff clay is a least permeable, and hence clay may be formed impermeable.

13. Define laminar and turbulent flow.

In laminar flow, each fluid particle travels along a definite path which never crosses the path of any other particle.

In Turbulent flow, the paths are irregular and twisting, crossing and recrossing at random.

14. What are the importances for the study of seepage of water?

1. Determination of rate of settlement of a saturated compressible soil layer.
2. Calculation of seepage through the body of earth dams, and stability of slopes.
3. Calculation of uplift pressure under hydraulic structure and there safety against piping.
4. Ground water flow towards well and drainage of soil

15. Define Darcy’s law.

Darcy’s law states that for laminar flow conditions in a saturated soil, the rate of flow or the discharge per unit time is proportional to the hydraulic gradient.

q = KiA

$$V = \frac{q}{A} = k_i$$

Where

q = discharge per unit time

A = Total cross-sectional area of soil mass, perpendicular to the direction of flow

i = hydraulic gradient

k = Darcy's Coefficient of permeability

v = Velocity of flow, or average discharge velocity.

If a soil sample of length L and cross-sectional area A, is subjected to differential head of water, h_1-h_2 the hydraulic gradient i will be equal to $\frac{h_1 - h_2}{L}$ and $q = k\frac{h_1 - h_2}{L}A$ When hydraulic gradient is unity, K is equal to v.

If a soil sample of Length L and cross-sectional area A, is subjected to differential head of water, h_1 –h_2, the hydraulic gradient I will be equal to $\frac{h_1 - h_2}{L}$ and $q = k\frac{h_1 - h_2}{L}A$

16. Define coefficient of permeability (or) permeability.

It is defined as the average velocity of flow that will occur through the total cross-sectional are of soil under unit hydraulic gradient. The coefficient of permeability is denoted as K. It is usually expressed as cm/sec (or) m/day (or) feet/day.

17. Define seepage velocity (or) Actual velocity.

The actual velocity (or) seepage velocity is defined as the rate of discharge of percolating water per unit cross-sectional area of voids perpendicular to the direction of flow.

18. State the factors affecting permeability.

i. Grain size
ii. Properties of the pore fluid
iii. Voids ratio of the soil
iv. Structural arrangement of the soil particle
v. Entrapped air and foreign-matter.
vi. Adsorbed water in clayey soils.

19. Mention the methods to determine the coefficient of permeability.

a. Laboratory methods
   i. Constant head permeability test
   ii. Falling head permeability test
b. Field methods
   i. Pumping – out tests
   ii. Pumping –in tests
c. Indirect methods
   i. Computation from grain size (or) specific surface

ii. Horizontal capillarity test
iii. Consolidation test dates.

20. Define capillary siphoning.

When the water level in the reservoir is corresponding to the flood level (H.F.L), the portion to the u/s of the dam will be saturated. The water level in the u/s pervious shell will be practically the same as the H.F>L. Due to capillarity, water will rise through a height h_c. If the top of the core is situated at a height $y < h_c$ above the H.F.L, , the capillary forces ill pull the water in descending part of the earth dam, and will slowly empty it. This process is known as capillary siphoning.

21. Define surface tension.

Surface tension of water is the property which exists in the surface film of water tending to contract the contained volume into a form having a minimum superficial area possible.

The surface tension (T_s) or coefficient of surface tension is approx equal to 72.8 dynes per cm (or) 0.728 x 10^{-6} KN/cm at 20°C.

The surface tension of water is more than double the surface tension for other common liquids. The surface tension for mercury is as high 2.45 x 10^{-6} KN/cm. The formation of curved meniscus around the other material inserted in water is due to the surface tension.

22. Explain the formation of meniscus:

When a solid or hollow tube, wet with water is partly inserted vertically in water, the molecules, due to attraction between the molecules of water and the material, climb the solid surface forming a curved meniscus adjacent to the walls of the tube or rod.

16 Marks Questions and Answers

1. Explain capillary rise?

The rise of water in the capillary tubes, or the fine pores of the soil, is due to the existence of surface tension which pulls the water up against the gravitational force. The height of capillary rise, above the ground water (or free water) surface depends upon the diameter of the capillary tube (or) fineness of the pores and the value of the surface tension. Fig.3 shows an enlarged view of a capillary tube inserted in water and the consequence capillary rise.

The formation of a concave meniscus will take place only if the inner walls of the tube are initially wet. If the walls are dry before insection, a convex meniscus depressed bellow the water is formed. The vertical components T_s Cos α of the surface tension force depends upon the angle of incidence d between the meniscus and the tube.

Let d = inner diameter of the tube

h_c = height of capillary rise

When the capillary tube is inserted in water, the rise of water will take place. When equilibrium has reached, water will stop moving further. If this equilibrium position, when the height of rise is h_c, the weight of column of water is equal to

$$\frac{\pi d^2}{4} h_c \gamma_\omega .$$

The vertical component of the reaction of meniscus against the inside circumference of the tube, supporting the above weight of water column is equal to

$$\pi d T_S \, Cos\,\alpha$$

Equating these two quantities at the equilibrium. $\frac{\pi d^2}{4} h_c \gamma_\omega = \pi d T_S \, Cos\,\alpha$

$$h_c = \frac{4 T_S \, Cos\,\alpha}{\gamma_\omega d}$$

The value of α will depend upon the initial conditions of the inner walls of the tube. If the tube is perfectly clean and wet, a semi-spherical meniscus will be formed. In that case, α will be zero, and maximum capillary rise will take place:
For water at 4° C

For water at 4° C

$$T_S = 75.6 \quad dynes \;/\; cm = \; 75.6 \times 10^{-8}\, KN / cm$$

$$\gamma_\omega = 9.807 KN / m^3$$

If d is expressed in cm, the above expression reduces to

$$\left(h_c\right)_{\max} = \frac{4 \times 75.6 \times 10^{-8}}{(9.807)\left(10^{-6}\right) d} = \frac{0.3084}{d} cm \qquad \text{(i)}$$

At 20 ° C,

$$T_S = 72.8 \quad dynes / cm = 72.8 \times 10^{-8}\, KN / cm$$

$$\gamma_\omega = 9.7876 \, KN / m^3$$

$$\left(h_c\right)_{\max} = \frac{4\times 72.8\times 10^{-8}}{\left(9.7896\times 10^{-6}\right)d} = \frac{0.2975}{d}\,cm \qquad \text{(ii)}$$

From equation (i) (ii), the height of capillary rise decreases with increase in temperature.

2. Explain capillary tension, capillary potential and soil suction.

At any height h above the water table, the stress is in water will be $-h\gamma_\omega$ (minus sign for tension). The maximum magnitude of the stress u will depend upon the radius R of the meniscus (shown in fig.4) the relation between the diameter d and the radius R is.

$$\frac{d}{2} = RCos\alpha$$

$$d = 2RCos\alpha$$

Substitute value in

$$h_c = \frac{4T_S Cos\alpha}{\gamma_\omega d}$$

$$h_c = \frac{4T_S Cos\alpha}{\gamma_\omega 2RCos\alpha}$$

$$h_c = \frac{4T_S}{\gamma\omega^2 R}$$

$\therefore$ u_c = maximum tension at the level of meniscus

$$\gamma_\omega hc = \gamma_\omega \frac{4T_S}{\gamma_\omega 2R} = \frac{2T_S}{R}$$

Thus the maximum tensile stress is inversely proportional to the radius of meniscus. When $\alpha = 0$, $R = d/2$, we have

$$\left(u_c\right)_{\max} = \frac{4T_S}{d}$$

The tensile stress, caused in water is called the capillary tension (or) the capillary potential. The capillary tension (or) capillary potential is the pressure deficiency, pressure reduction (or) negative pressure in the pore water by which water is retained in a soil mass. It decreases linearly from a maximum value of $h_c \gamma_\omega$ at the level of the meniscus to zero value at the free water surface. The pressure deficiency in the held water is also termed as soil suction (or) suction pressure.

Capillary pressure:

The capillary pressure distribution is rectangular unlike the triangular distribution of capillary tension. The magnitude of the pressure is the same at all heights above the free water surface.

The capillary pressure, transferred from grain to grain may be also called intergranular (or) contact (or) effective pressure.

3. Define Non-uniform meniscus and explain stress condition in soil.

If the meniscus is not of uniform curvature, but R_1 and R_2 are the radii of curvature in two orthogonal planes, the height of capillary rise is given by

$$h_c = \frac{0.3084}{0.00857} = 36\,cm$$

Stress conditions in soil:

Effective and Neutral pressures

The total stress (or) unit pressure (σ) is the total load per unit area. This pressure may; be due to

i. Self –weight of soil (saturated weight, if the sol is saturated)
ii. Over –burden on the soil

1 The total pressure consist of two distinct components:

i. Intergranular pressure (or) effective pressure
ii. Neutral pressure (or) pore pressure

Effective pressure (σ^1) is the pressure transmitted from particle through this point of contact through the soil mass above the plane.

The neutral pressure (u) (or) the pore pressure (or) the pore water pressure is the pressured transmitted through the pore fluid.

Since the total vertical pressure at any plane is equal to the sum of the effective pressure and the pore pressure.

$$\sigma = \sigma^1 + u$$

At any plane, the pore pressure is equal to piezometric head h_ω times the unit weight of water, (i.e)

$$u = h_\omega \gamma_\omega$$

To find the value of effective pressure, different conditions of soil water system is considered.

1. Submerged soil mass
2. Soil mass with surcharge
3. Saturated soil with capillary fringe

1. **Submerged soil mass:**

Fig.5 shows a saturated soil mass of depth Z, submerged under water of height Z_1 above its top level. If a piezometric tube is inserted level AA1, water will rise in it upto level cc.

Now, total pressure at AA is given by

$$\sigma = Z\gamma_{sat} + Z_1\gamma_\omega$$

Also, pore pressure $u = \gamma_\omega h_\omega$

$$\sigma^1 = \sigma - u$$

$$= Z\gamma_{sat} + Z_1\gamma_{sat} - h_{sat}\gamma_{sat}$$

$$= Z\gamma_{sat} + Z_1\gamma_\omega - (Z + Z_1)\gamma_\omega$$

$$= Z\gamma_{sat} + Z_1\gamma_\omega - Z\gamma_\omega - Z_1\gamma_\omega$$

$$\sigma^1 = Z(\gamma_{sat} - \gamma_\omega) = Z\gamma^1$$

Hence the effective pressure is equal to the thickness of the soil multiplied y the submerged weight of soil. It does not depend upon the height Z_1, of the water column. Even if Z_1 reduces to zero, σ^1 will remain equal to $Z\gamma^1$ so long as the soil mass above AA remains fully saturated. At BB, the total pressure is equal to the water pressure $Z_1\gamma\omega$ and hence the effective pressure is zero.

2 **Soil mass with surcharge**:

Let us now consider a moist soil mass of height Z_1 above a saturated mass of height Z. Soil mass supports a surcharge pressure of intensity ‘q’ per unit are.

At the level AA, the pressure is:

$$\sigma = q + Z_1\gamma + Z\gamma_{sat} \qquad \because Z(\gamma_{sat} - \gamma\omega) = Z\gamma^1$$

$$u = h_\omega \gamma_\omega = Z\gamma_\omega$$

$$\sigma^1 = \sigma$$

$$= q + Z_1\gamma + Z\gamma_{sat} - Z\gamma_\omega \qquad \because Z(\gamma_{sat} - \gamma\omega) = Z\gamma^1$$
$$= q + Z_1\gamma + Z\gamma^1$$

At the plane BB

$$\sigma = q + z_1\gamma$$

$$u = h_\omega \gamma_\omega = 0$$

$$\sigma^1 = \sigma - u$$

$$\sigma^1 = \sigma$$

$$\therefore \qquad \sigma^1 = q + z_1\gamma$$

At the plane CC, effective pressure = total pressure = q

3. Saturated soil with capillary fringe.

Fig.7 shows a saturated soil mass $Z_1\gamma\omega$ of height z. Above this, there is a soil mass of height, Z1 saturated by capillary water.

If we insert a piezometric tube at AA, water will rise to a height corresponding to the free water level BB.

The capillary pressure (or) compressive pressure on the soil grains. This pressure is also inter-granulate and is effective in reducing the rods ratio of the soil mass. This compressive pressure is equal to $h_c\gamma_\omega = Z_1\gamma_\omega$ in this case.

Hence at level AA,

$$\sigma = Z\gamma_{sat} + Z_1\gamma_{sat}$$

$$\therefore \sigma^1 = \sigma - u \qquad u = Z\gamma\omega \qquad u = h\omega$$

$$u = h\omega\gamma\omega$$

$$= Z\gamma_{sat} + Z_1\gamma_{sat} - Z\gamma_{\omega}$$

$$= Z_1\gamma_{sat} + Z(\gamma_{sat} - \gamma_{\omega})$$

$$\sigma^1 = Z_1\gamma_{sat} + Z\gamma^1$$

Similarly at the level BB,

$$\sigma^1 = Z_1\gamma^1 + Z_1\gamma_{sat}$$

$$= Z_1(\gamma_{\omega} + \gamma^1) \qquad \gamma^1 = \gamma_{sat} - \gamma_{\omega}$$

$$\gamma_{sat} = \gamma^1 + \gamma_{\omega}$$

$$\sigma^1 = Z_1\gamma_{sat}$$

Finally at CC,

σ^1 = Capillary pressure = $Z_1\omega$

The effective pressure distribution diagram is shown in fig.

At any depth x below the level CC

$$\sigma = x.\gamma_{sat}$$

u = - [Pressure due to weight of water hanging below that level]

$$= -(Z_1 - x)\gamma\omega$$

$$\sigma^1 = \sigma - u = x\gamma_{sat} + (Z_1 - x)\gamma_{\omega}$$

$$= x(\gamma_{sat} - \gamma\omega) + Z_1\gamma_{\omega}$$

$$\sigma^1 = x\gamma^1 + Z_1\gamma_{\omega}$$

4. The water table in a certain area is at a depth of 4m below the ground surface. To a depth of 12m, the soil consists of every fine sand having an average voids ratio of 0.7. Above the water table the sand has an average degree of saturation of 50%. Calculate the effective pressure on a horizontal plane at a depth 10 meters hollow the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillarity up to a height of 1m above the water table? Assume G = 2.65

Solution:

Height of sand layer above water table = Z_1 = 4 m

Height of saturated layer above water table = 12 - 4 = 8 m

Depth of point X, where pressure is to be computed = 10 m

Height of saturated layer above X = Z_2 = 10 - 4 = 6 m

Now

$$\gamma d = \frac{G\gamma\omega}{1+e} = \frac{2.65 \times 9.81}{1+0.7} = 15.29 KN/m^3$$

i. For sand above water table:-

$$e = \frac{\omega G}{Sr}$$

$$\omega = \frac{eS_r}{G} = \frac{0.7 \times 0.5}{2.65} = 0.132$$

$$\gamma_1 = \gamma d(1+\omega) = 15.29 \times 1.132 = 17.31 KN/m^3$$

ii. For saturated sand below water table

$$\omega_{sat} = \frac{e}{G} = \frac{0.7}{2.65} = 0.264$$

$$\gamma_2 = \gamma d(1+\omega_{sat})$$

$$15.29(1+0.264)$$

$$\gamma_2 = 19.33 KN/m^3$$

$$\gamma_2^{\,1} = 19.33 - 9.81 = 9.52\ KN/m^3$$

Effective pressure at X

$$\sigma = Z_1\gamma_1 + Z_2\gamma_2$$

$$\sigma = 4 \times 17.31 + 6 \times 19.33$$

$$= 185.22 \text{ KN/m}^2$$

$$u = h_\omega \gamma_\omega = 6 \times 9.81 = 58.86 KN/m^2$$

$$\sigma^1 = \sigma - u = 185.22 - 58.86 = 126.36 KN/m^2$$

Effective stress at x after capillary rise

$$\sigma^1 = 3\gamma_1 + (6+1)\gamma_2{}^1 + h_c\gamma_\omega$$

$$= (3 \times 17.31) + (7 \times 9.52) + (1 \times 9.81)$$

$$= 128.38 \text{ KN/m}^2$$

Increase in pressure

$$= 128.38 - 126.36 = 2.02 \text{ KN/m}^2$$

Result:

i. Effective pressure at a depth of 10m = 128.38 KN/m^2

ii. Increase in pressure = 2.02 KN/m^2

5. A 10m thick bed of sand is underlain by a layer of clay of 6 m thickness. The water table witch was originally at the ground surface is lowered by drainage to a depth of 4m , where upon the degree of saturation above the lowered water table reduces to 20%. Determine the increase in the magnitude of the vertical effective pressure at the middle of the day layer due to lowering of water table, the saturated unit weights of sand and clay are respectively 20.6 KN/m^3 and 17.6 KN/m^3 and the dry unit weight of sand is 16.7 KN/m^3.

Solution:

i) Before lowering the water table, the pressures at the middle of the clay layer are

$$\sigma = (10 \times 20.6) + (3 \times 17.6)$$

$$= 258.8 \text{ KN/m}^2$$

$$u = 13 \times 9.81 = 127.53 \text{ KN/m}^2$$

$$\sigma^1 = \sigma - u$$

$$= 258.8 - 127.53 = 131.27 \text{ KN/m}^2$$

ii) After lowering the water table, the unit weight of sand is given by

$$\gamma = \gamma_d + S_r(\gamma_{sat} - \gamma_d)$$

= 16.7+0.2 (20.6-16.7)
= 17.48 KN/m^3

$$\sigma = (4\times17.48)+(6\times20.6)+(3\times17.6)$$

= 246.32 KN/m^2

u = 9 x 9.81 = 88.29 KN/m^2

σ^1 = 246.32 - 88.29 = 158.03 KN/m^2

$\therefore$ Increase in effective pressure

= 158.03 - 131.27
= 26.76 KN/m^2

6. The water table in a deposit of sand 8 m thick is at a depth of 3m below the surface. Above the water table, the sand is saturated with capillary water. The bulk density of sand is 19.62 KN/m^3. Calculate the effective pressure of 1m, 3m and 8m below the surface. Hence plot the variation of total pressure, neutral pressure and effective pressure over the depth of 8 m.

Solve:

a. Stresses at D, & 8 m below ground:

If we insert a piezometric tube at D, water will rise through a height $h\omega$ = 5m in it.

σ = (3+5) γ_{sat}
= 8 x 19.62
σ = 156.96 KN/m^2

u = $h_\omega \gamma_\omega$

= 5 x 9.81 = 49.05 KN/m^2

$$\sigma^1 = \sigma - u = 156.96 - 49.05 = 107.91 KN/m^2$$

b. Stresses at C, 3m below ground level:

$$\sigma = 3\gamma_{sat} = 3 \times 19.62 = 58.86 KN/m^2$$

u = 0

$$\sigma^1 = \sigma - u$$

$$\sigma^1 = 58.86\ KN/m^2$$

c Stress at A, at ground level

$\sigma = 0$

$$u = -h_c \gamma_\omega = -3(9.81) = -29.43\ KNm^2$$

$$\sigma^1 = \sigma - u = 29.43\ KN/\ m^2$$

d. Stresses at B, 1m below ground level

$\sigma = 1\gamma_{sat}$ = 1 x 19.62 = 19.62 KN/m^2

$u = -2\gamma_\omega$ = -2 x 9.81 = -19.62 KN/m^2

(i.e.) Pressure due to weight of water hanging below that level

$$\sigma^1 = (\sigma - u) = 19.62 - (-19.62)$$

$$= 19.62 + 19.62$$

$$\sigma^1 = 39.34\ KN/m^2$$

The total stress, effective stress and pore pressure distribution are shown in fig.

7. Describe Poiseuille's Law of flow through capillary tube.

Solution:

The relationship governing the laminar flow of water through capillary tube is known as Poiseuiller's law. The fig.8.a. Shows a capillary tube, of length L and radius R. Velocity distribution is shown in fig.8.b. At any radial distance from the centre, the velocity is V and the velocity gradient (i.e.)

Space rate of change of velocity $u - \frac{dv}{dr}$. The unit shear at the top and bottom of the cylinder of water of radius r is given by

$$\tau = -\eta \frac{dv}{dr} \qquad (1)$$

Where η is the co-efficient of viscosity

If the tube is subjected to a head of water h_1 at one end, and h_2 ($h_1>h_2$) of the other end, flow will take place and various forces acting on the cylinder of water at any radius r. Since the tube is in equilibrium, the sum of all forces acting on the cylinder must be zero. Hence

$$h_1\gamma\omega\left(\pi r^2\right)-h_2\gamma\omega\left(\pi r^2\right)-\tau\left(2\pi rL\right)=0$$

$$\tau\left(2\pi rL\right)=\left(h_1-h_2\right)\pi r^2\gamma\omega$$

$$dv=-\frac{h_1-h_2}{L}\cdot\frac{\gamma\omega}{2\eta}rdr$$

Replacing h_1-h_2 by h (net head causing flow) and integrating,

$$V=\frac{-h\gamma\omega}{4\eta L}r^2+C$$

At r = R, v = 0 $\therefore$ $C=\frac{h\gamma\omega}{4\eta L}R^2$

Hence

$$v=\frac{h\gamma\omega}{4\eta L}\left(R^2-r^2\right) \qquad (2)$$

This is the law of variation of velocity.

The quantity of water flowing in the thin cylindrical sheet dr thick, is given by

$$d_q=\left(2\pi rdr\right)v=\frac{h\gamma\omega}{4\eta L}\left(R^2-r^2\right)2\pi r\,dr$$

$\therefore$ Total quantity of water flowing in the capillary tube, per unit time is

$$q=\frac{h\gamma\omega}{4\eta L}2\pi\int_0^R\left(R^2-r^2\right)r\,dr$$

$$q = \frac{h\gamma\omega}{8\eta L}\pi R^4$$

Replacing $\frac{h}{L} = i =$ hydraulic gradient

$$q = \frac{\gamma_\omega \pi R^4}{8\eta} i \qquad (3)$$

If a is the area of the tube, average velocity is given by

$$V_{av} = \frac{q}{a} = \frac{q}{\pi R^2} = \frac{\gamma\omega R^2}{8\eta} i \qquad (4)$$

It is the Poiseuille's laws in which the velocity of flow during laminar flow varies as the first power of the hydraulic gradient.

Effect of shape of the capillary tube:

The above equations are valid for circular capillary tube only. The velocity is generally designated in terms of the hydraulic radius R_H, which is defined as the ratio of area to the wetted perimeter.

For circular tube

$$R_H = \frac{\pi R^2}{2\pi R} = \frac{R}{2}$$

$R = 2R_H$

$$\therefore \quad q_{cir} = \frac{1}{2}\frac{\gamma_\omega RH^2}{\eta} ia \qquad (5)$$

For closely spaced parallel plates

$$q_{pha} = \frac{\frac{1}{3}\gamma_\omega RH^2}{\eta} ia \qquad (6)$$

[redacted]

$$q = Cs\frac{\gamma_\omega RH^2}{\eta} ia \qquad (7)$$

Where C_s = shape constant

For irregular capillary voids

a = area of flow passage

= nA

$$q = \left(Cs\frac{\gamma_\omega RH^2}{\eta} n \right) iA$$

$$q = \left(Cs\frac{\gamma_\omega RH^2}{\eta} \frac{e}{1+e} \right) iA \qquad (8)$$

Hydraulic mean radius in soil pores:

If Vs is the volume of solids in a soil mass having voids ratio e, the volume of the flow channel (=AL) will be eVs. The total surface area of flow channel (=PL) is equal to the total surface area A_S of the soil grains.

$$R_H = \frac{AL}{PL} = \frac{eVs}{As}$$

Let Ds = diameter of the spherical grain

$$\frac{Vs}{A_S} = \frac{\frac{1}{6}\pi D_s^3}{\pi Ds^2} = \frac{Ds}{6}$$

$$R_H = \frac{Ds}{6} e$$

$$\therefore \quad q = \left(Cs.\frac{\gamma_\omega}{\eta}.\frac{e^3}{1+e}\frac{Ds^2}{36} \right) iA$$

[redacted]

[redacted]

$$q = \left(C.\frac{\gamma_{\omega}}{\eta}.\frac{e^{3}}{1+e} Ds^{2} \right) iA$$

Where, C is a new shape constant.

8. Calculate the co-efficient of permeability of a soil sample, 6 cm in height and 50 cm^2 in cross-sectional area, if a quantity of water equal to 430 ml passed down in 10 min. Under an effective constant head of 40 cm.

On oven-drying, the test specimen has mass of 498 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.

Solution:

Given,

Q = 430 ml ; t = 10 x 60 = 600 seconds

A = 50 cm^2 : L = 6 cm ; h = 40 cm

From the equation for constant head permeability test

$$K = \frac{430}{600} \times \frac{6}{40} \times \frac{1}{50}$$

K = 2.15 x 10^{-3} cm/sec

= 2.15 x 10^{-3} x 864 = 1.86 m/day

(Since 1cm/sec=864 m/day)

Now

$$V = \frac{q}{A} = \frac{430}{600 \times 50} = 1.435 \times 10^{-2}\, cm/\sec$$

Now

$$\rho_d = \frac{498}{50 \times 6} = 1.66 g/cm^{3}$$

$$e = \frac{G\rho_{\omega}}{\rho_d} - 1 = \frac{2.65 \times 1}{1.66} - 1 = 0.595$$

[redacted]

[redacted]

$$n = \frac{\rho}{1+\rho} = \frac{0.595}{1.595} = 0.373$$

$$v_s = \frac{v}{n} = \frac{1.435\times 10^{-2}}{0.373} = 3.85\times 10^{-2}\, cm/\text{sec}.$$

9. In a falling head permeameter test, the initial head (t = 0) is 40 cm. The head drops by 5 cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20cm. If the sample is 6 cm is height and 50 cm^2 in cross-sectional area, calculate the coefficient of permeability, taking area of stand pipe = 0.5 cm^2

Solution:

In a time interval t = 10 minutes, the head drops from initial value of $h_1 = 40$ to $h_2 = 40 - 5 = 35$ cm

From the equation for falling head permeameter

$$K = 2.3\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$$

$$t = \frac{2.3aL}{AK}\log_{10}\frac{h_1}{h_2} = m\log_{10}\frac{h_1}{h_2}$$

Where $m = \frac{2.3aL}{AK} =$ Constant for the set up

$$10 = m\log_{10}\frac{40}{35}$$

$$m = \frac{10}{\log_{10}\frac{40}{35}} = \frac{10}{0.058} = 172.units$$

$$t = m\log_{10}\frac{h_1}{h_2}$$

$$= 172.5\log_{10}\frac{h_1}{h_2}$$

[redacted]

Now, let the time interval required for the head to drop from initial value of $h_1$40cm to a final value of h_2 = 20cm, be t minutes.

$$t = 172.5 \ \log_{10} \frac{40}{20} = 172.5 \times 0.301 = 5.19 \min utes$$

Again $m = \dfrac{2.3aL}{AK} = 172.5\, Units$

$$K = \frac{2.3aL}{4 \times 172.5} Cm/\min$$

(Since t used to compute m was in minutes)

$$K = \frac{2.3 \times 0.5 \times 6}{50 \times 172.5 \times 6} Cm/\sec.$$

$$= 1.33 \times 10^{-5} Cm/\sec.$$

10) a) What is seepage force or seepage pressure?

By virtue of the viscous friction exerted or water flowing through soil pores, or energy transfer is effected between the water and soil. The force corresponding to this energy transfer is called the seepage force or seepage pressure.

Seepage pressure $P_s = h\gamma\omega$

$$= \frac{h}{z} z\gamma\omega$$

$$= i.z.\gamma\omega$$

Z - Thickness of soil mass
i - Hydraulic gradient

Seepage force J = Ps. A
To total cross-sectional force = $iZ\gamma\omega.A$

Seepage force per unit volume $i = \dfrac{iz\gamma\omega A}{ZA}$

$$= i\,\gamma\,\omega$$

$$\sigma^1 = Zy^1 + ps = Zy^1 \pm iz\gamma\omega$$

[redacted]

10) b) What is upward flow or Quick condition? Explain in brief?

When flow takes place in an upward direction. The seepage pressure also acts in the upward direction and the effective pressure reduced. If the seepage pressure becomes equal to the pressure due to submerged weight of the soil, the effective pressure is reduced to zero. In such a case, a cohesonless soil loses all its shear strength, and the soil particles have a tendency to move up in the direction of flow. This phenomenon of lifting of soil particles have a tendency to move up on the direction of flow. This phenomenon of lifting of soil particles is called quick condition, boiling condition or quick sand.

The tuning the quick condition,

$$\sigma^1 = ZY^1 - Ps = 0$$

$$Ps = ZY^1 (or) iZ\gamma\omega = zY^1$$

From which

$$i = i_c = \frac{\gamma^1}{\gamma_\omega} = \frac{G-1}{1+e}$$

The hydraulic gradient at such a critical state is called the critical hydraulic gradient.

For loose deposits of sand or silt, if roids ratio e is taken as 0.67 and G as 2.67, the critical hydraulic gradient works out to be unity.

It should be noted that quick sand is not a type of sand but a flow condition occurring within acohesionless soil when its effective pressure is reduced to zero due to upward flow of water.

Figure.9. shows a set-up to demonstrate the phenomenon of quick sand. Water flows in an upward direction through a saturated soil sample of thickness z under a hydraulic head h.

This head can be increased or decreased by moving the supply tank in the upward or decreased by moving the supply tank in the upward or downward direction. When the soil particles are in the state of critical equilibrium, the total upward force of the bottom of the soil becomes equal to the total weight of all the materials above the surface considered.

Equating the upward and downward forces at the level a –a, we have

$$(h + Z)\gamma\omega = Z\gamma_{sat}.A$$

$$\frac{h}{z} = i_c = \frac{\gamma^1}{\omega} = \frac{G-1}{1+e}$$

$$\therefore \quad h\gamma\omega = z(y_{sat} - \gamma\omega)$$

[redacted]

$= z\gamma^1$

11.Explain the Laplace equation for two dimensional flow.

Assumption

1. The saturated porous medium is compressible. The size of the pore space doesn't change with time, regardless of water pressure.

2. The seeping water flows under a hydraulic gradient which is due only to gravity head loss, or Darcy's law for flow through porous medium is valid.

3. There is no change in the degree of saturation in the zone of soil through which water seeps and quantity of water flowing into any element of volume is equal to the quantity which flows out in the same length of time.

4. The hydraulic boundry conditions of any entry and exit are known

5. Water is incompressible.

Consider an element of soil of size Δx, Δy and of unit thickness perpendicular to the plane of the paper Let V_x and V_y be the entry velocity components in X and Y directions.

Then $\left(V_s + \frac{\partial Vx}{\partial x}.0x\right)$ and $\left(V_y + \frac{\partial Vy}{\partial y}.\Delta y\right)$ will be the corresponding velocity components of the exit of the element.

According to assumption 3 stated above. The quantity of water entering the element is equal to quantity of water leaving it.

$$V_x(\Delta y.1) + V_y(\Delta x.1) = \left(V_x + \frac{\partial Vx}{\partial x}\Delta x\right)(\Delta y.1) + \left(V_y + \frac{\partial Vy}{\partial y}\Delta y\right)(\Delta y.1)$$

From which

$$\frac{\partial Vx}{\partial x} + \frac{\partial Vy}{\partial y} = 0$$

This is the continuity equation according to assumption

$V_x = V_x$, $\quad ix = Vx\frac{\partial h}{\partial x}$ And

[redacted]

$$V_y = V_y, \qquad iy = k_y \frac{\partial h}{\partial y}$$

Where,

h = hydraulic head under which water flows.

V_x & V_y = Co-efficient of permeability in x and y directions.

Substituting these

$$\frac{\partial^2 (k_x h)}{\partial x^2} + \frac{\partial^2 (k_y h)}{\partial y^2} = 0$$

For an isotropic soil,

$$v_x = v_y = k(say) \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Substituting $\phi = k_h$ = Velocity potential,

We get $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

This is the Laplace equation of flow in the dimensions.

<u>Velocity potential (ϕ)</u>

The velocity potential ϕ may be defined as a scalar function of space and time such that its derivative with respect to any direction gives the fluid velocity in that direction. This is evident, since, we have ϕ =kh

$$\frac{\partial \phi}{\partial x} + k \frac{\partial h}{\partial x} = Kix = V_x$$

Similarly,

$$\frac{\partial \phi}{\partial x} = k \frac{\partial h}{\partial x} = Kiy = V_y$$

The solution of equation

[redacted]

$$\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0\right]$$

Can be obtained by

i. Analytical methods
ii. Graphical method
iii. Experimental methods

The solution gives two sets of curves known as 'equipotential lines' and 'stream lines' (or flow lines), mutually orthogonal to each other, as shown in figure.10.

The equipotential lines represent contours of equal head (potential). The direction of seepage is always perpendicular to the equipotential lines. The path along which the individual particles of water seep through the soil are called stream lies or flow lines.

12. a) Explain properties of flow nets.

1. The flow lines and equpotential lines meet at right angles to one another.
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3. The quantity of water flowing through each flow channel is the same, similarly, the same potential.
4. Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.
5. In a homogeneous soil, every transition in the shape of the curves is smooth, being either elliptical or parabolic in shape.

12. b) Explain flow net By Electrical analogy

The Darcy's law governing the flow of water through soil is analogous to ohms law governing the flow of electric current through conductors.

Thus, the solutions to seepage problems can be obtained with electric models which have the same geometric shape as the soil through which the water flows.

The seepage medium is replaced by an electric conductor consisting of water with some salt or dilute hydrochloric acid. The boundary equipotential lines are mode of copper.

The boundary low lines are simulated by non conducting strips such as ebonite or Perspex etc. A alternating voltage (generally of value 5 to 20 volts.) is applied across the boundry equipotential strips.

A potential divider is connected in parallel with alternating current source.

From figure11., the electric analogy way, the model and the completer circuit diagram, for study of seepage through an earth dam.

To determine a line of contour of equal potential, the potentiometer is adjusted to a percentage of the total voltage drop, and the probe of a galvanometer is used to find the corresponding balance pints (Null points) or the model.

Changes in the co-efficient of permeability in soil zones in the seepage analogue are simulated by changes in the electric conductivity co-efficient in the model.

When once the equipotential lines are obtained, orthogonal flow lines confirming to the boundry conditions are then drawn as in the graphical method.

From figure, it shows a typical flow net for steady seepage case for an earth dam waving the same foundation material as that of the body of the dam.

13. Applications of flow net: Explain in brief

i. Determination of seepage
ii. Determination of hydrostatic pressure
iii. Determination of seepage pressure
iv. Determination of exit gradient

i.Determination of seepage

The portion between any two successive flow lines is at flow channel. The portion enclosed two successive equipotential lines and successive flow lines are known as field.

Let b and l be the width and length of the field.

Δ h = head drop through the field

Δ q = discharge passing through the flow channel

H = Total hydraulic head causing flow = difference between upstream and downstream weeds.

ii.Determination of hydrostatic pressure.

The hydrostatic pressure at any point within the soil mass is given by

$$u = h_{\omega}\gamma_{\omega}$$

Where, u = hydrostatic pressure

hω = Piezometric head.

[redacted]

The hydrostatic pressure in terms of piezometric head h_ω is calculated from the following relation.

$$h_\omega = h - z$$

Where,

h – Hydraulic potential at the point under consideration

Z – Position weed of the point above datum, considered positive upwards.

Then, from Darcy's law of flow through soils.

$$\Delta a = k.\frac{\Delta h}{l}(b \times l)$$

x/d = Total x/0 potential drops in the completer flow net,

$$\Delta h = \frac{H}{xld}$$

Hence,

$$\Delta q = k.\frac{H}{Nd}\left(\frac{b}{l}\right)$$

The total discharge through the complete flow net is given by

$$q = \Sigma \Delta q = k.\frac{H}{Nd}\left(\frac{b}{l}\right).\lambda / f$$

$$= k.H\,\frac{\lambda / f}{\lambda / d}\,\frac{b}{l}$$

x/f = Total number of flow channels in the net.

Where,

The field is square, hence b = l

$\therefore$ Thus, $$b = k.H\,\frac{\lambda / f}{\lambda / d}$$

This is the required expression for discharge passing through a flow net and is valid for isotropic soils in which $k_x = k_y = K$

iii.Determination of seepage pressure

The hydraulic potential h at any point located after N potential drops, each of value Δh is

[redacted]

given by

$$b = H = -n\Delta h$$

The seepage pressure of any point the hydraulic potential or the balance hydraulic head multiplied by the unit weight of water,

$$Ps = h\gamma_{\omega} = (H - n\Delta h)\gamma\omega$$

The pressure acts in the direction flow

iv.Determination of exit gradient.

The exit gradient is the hydraulic gradient of the downstream end of the flow line where the percolating water leaves the soil mass and emerges into free water at the downstream.

The exit gradient can be calculated from the following expression, in which Λh represents the potential drop and l the average length of last field in the flow net alt the exit end.

$$ie = \frac{\Delta h}{l.}$$

UNIT – III
STRESS DISTRIBUTION

Stress distribution in soil media – Boussinque formula- Stress due to line load and circular and rectangular area – Approximate methods – Use of influence charts – Westergaard equation for point load – Components of settlement – Terzaghi's one dimensional consolidation theory- Governing differential equation – Laboratory consolidation test – Field consolidation curve – NC and OC clays- Problems on final and time rate of consolidation**.**

Two mark questions

1. What are the assumption are made in the Boussinque equations.

1 The soil mass is homogenous, that is all its constituent parts (or) elements are

similar and it has identical properties at every point in it in identical directions.

2 Te soil mass is an elastic medium for which the modulus of elasticity E is constant.

3 The soil mass is "Isotropic" that is it has identical elastic properties in all directions through any point of it.

4 The soil mass is semi infinite that is, it extends infinitely in all directions below a level surface.

2.What are the Symbol to be Used For Stress Distribution?

The total stress field at point within a soil mass loaded at its boundary consists of nine stress components given below.

$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

These, nine stress components as given by this group of square matrix of stress are the components of a mathematical entry called the stress tensor of a symmetrical matrix relative to its main diagonal (upper left (or) lower right. The main diagonal elements of the stress tensor are the normal stress components and the off diagonal elements are shear stress out of the nine stress components indicated above, there are three independent shear components making the total unknowns to be equal to six. The corresponding nine strain components are given by the following strain tensor.

$$\begin{bmatrix} \in_x & \frac{1}{2}\gamma_{xx} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \in_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \xi_z \end{bmatrix}$$

Where and ε denotes the linear or direct strain and γ denotes the shearing strain .

3. Write about the Pressure Distribution Diagrams Types.

By means of Boussinesq's stress distribution theory, the following vertical pressure distribution diagrams can be prepared.

1. Stress isobar (or) isobar diagram
2. Vertical pressure distribution on a horizontal plane
3. Vertical pressure distribution on a vertical line.

4. What Is Iso-Bar?

An Isobar is a curve or counter connecting all points below the ground surface of equal vertical pressure on a given horizontal plane is the some in all directions at points located at equal radial distances around the axis of loading

5. Define the pressure bulb.

[redacted]

The some in a loaded soil mass bounded by on isobar of given vertical pressure. intensity is called a "pressure bulb".

6. Define Contact Pressure?

Contact pressure defined as the vertical pressure acting at the the surface of contact between the base of footing and the underlying soil mass.

7. What Is Compressibility?

When the compressive load is applied to soil mass, a decrease in its volume takes places. The decrease in the volume of soil mass under stress is known as compression and the property of soil mass compressibility.

8. What is consolidation?

Every process involving a decrease in the water content of a saturated soil without replacement of the water by air is called process of consolidation.

9. Define the Co-efficient of Compressibility.(a_v)

The co-efficient of compressibility is defined as the decrease in voids per unit increase of pressure.

$$a_v = \frac{-\Delta_e}{\Delta\sigma^1} = \frac{e_0 - e}{\sigma^1 - \sigma_0^1}$$

10. Define of volume change (m_v)

The co-efficient of volume change or the co-efficient of volume compressibility is defined as the change in volume of a soil mass per unit of initial volume due to a given increase in the pressure.

11. Write short notes on consolidation of undisturbed specimen?

Soil deposits may be divided into three classes as regards to the consolidation history; pre consolidation normally consolidated and under consolidated. Clay is said to be pre compressed pre consolidated or over consolidated.

If it has ever been subjected to a pressure in excess of it present overburden pressure the temporary overburden pressure to which a soil has been subjected and under which it got consolidated is known as pre-consolidation pressure.

A soil may have been subjected during metal away by other geologic over burden and structural level which to longer exist now. A soil which is not fully consolidated existing over burden called an under consolidation.

12. How do you determine the pre-consolidation pressure?

To find the pre consolidation pressure on disturbed sample of clay is consolidated in the laboratory and the pressure voids ratio relationship is plotted on a semi-log plot.

The initial portion of the curve is that and assembles the recompression curve of a remolded specimen. The lower portion of the curve which is a straight line is the laboratory virgin curve.

The approximate value of the pre-consolidation pressure σ_P^1 may be determined by the following empirical method of A casagrande. The point A of maximum curvature selected and horizontal line AB: is drawn. A tangent AC is drawn to the curve and bisector AD, bisecting angle BAC is drawn.. The straight portion of the virgin curve is extended back to meet the

[redacted]

bisector AD in P. The point P corresponds to the pre consolidation pressure σ_P^1.

13. What are the assumption are made in the Terzaghi's theory of one-dimensional consolidation.

1 soil homogenous and fully saturated
2 Soil particles and water are incompressible.
3 Deformation of the soil is due entirely to change in volume
4 Darcy's law for the velocity of flow of water thorough soil is perfectly valid.
5 Coefficient of permeability is constant during consolidation
6 Load is applied deformation occurs only in direction
7 The change in thickness of the layer during consolidation is insignificant.

16 MARKS QUESTIONS AND ANSWERS.

1. Explain the Stresses Due To Self Weight of soil.

We shall consider the stresses within a soil mass due to its own weight stresses due o self weight are some times known as geostatic stresses. Let us take the soil mass to be bounded by the horizontal plane (ground surface) xy and the Z-axis be directed down wards. Under this condition, the soil mass is said to be semi-infinite where there is no external loading, the ground plane becomes or principal plane since it is devoid of ay shear loading. From the symmetry and the orthogonally plane since it is devoid of any shear loading. From the symmetry and the orthogonally of principal planes, are can conclude that all the horizontal and vertical planes will be devoid of shear stress, so that within soil mass. $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$. Substituting this in the equilibrium equations, we get $\sigma_z = \gamma_z$ →3.1

Where;

γ = unit weight of soil and

σ_z= vertical stress at a point within and soil mass, situated at a depth 'Z' below the ground surface.

Similarly, from compatibility equations in terms of stresses (for a three dimensional case.)

One obtains, $$\sigma_x = \sigma_y = \frac{\mu}{1-\mu}\gamma_z = K_o\gamma.Z$$ → 3.2

μ → Poisson's ratio.

$K_0 = \dfrac{\mu}{1-\mu}$ = Coefficient of lateral pressure at rest.

Thus, equation 3.1 and 3.2 give stress components at a point situated at depth Z below the ground surface, due to self weight of the soil mass above it.

At a certain point within the soil mass, the stresses components due to both these loading. (i.e self weight and surface loadings) can be found separately and then added algebraically to get then final stresses at the points. In the following articles, we shall therefore discuss the stress distribution due to surface loading alone.

2. Explain The Concentrated Force By Boussinesq Equations:

[redacted]

Boussinesq (1885) solved the problem of stress distribution in soils due to concentrated load acting at the ground surface by assuming a suitable stress function. The following assumptions are made in the solutions by the theory of elasticity.

1. The soil mass is homogenous, that is all its constituent parts (or) elements are similar and it has identical properties at every point in it in identical directions.
2. Te soil mass is an elastic medium for which the modulus of elasticity E is constant.
3. The soil mass is "Isotropic" that is it has identical elastic properties in all directions through any point of it.
4. The soil mass is semi infinite that is, it extends infinitely in all directions below a level surface

Let a point load QL (single concentrated vertical load) act a t the ground surface at a point. 'O' which may be taken as the origin of the x, y and Z axes as shown. Let us find the stress components at a point P in the soil mass, having coordinates x, y and Z or having a radial horizontal distance 'r' and vertical distance 'Z' form the point O.

Using the logarithmic stress function Boussinesq showed that the polar radial stresses may be expressed as.

$$\sigma_R = \frac{3}{2}\frac{Q}{\pi}\frac{Cos\beta}{R^2} \qquad \rightarrow 3.1$$

where R = Polar radial coordinate of point

$$P = \sqrt{r^2 + Z^2} = \left(x^2 + y^2 + z^2\right)^{1/2} \text{ and } Cos\beta = \frac{Z}{R}$$

In the cylindrical co ordinates the corresponding vertical stress σ_Z and tangential stress τ_{rz} are given by,

$$\sigma_z = \sigma_R Cos^2\beta = \frac{3}{2}\frac{Q}{\pi}\frac{Cos^3\beta}{R^2} = \frac{3}{2}\frac{Q}{\pi}\frac{Z^3}{R^5} \qquad \rightarrow 3.2$$

$$\sigma_z = \frac{3Q}{2\pi}\frac{Z^3}{\left(r^2 + z^2\right)^{5/2}} = \frac{3Q}{2\pi Z^2}\left[\frac{1}{1+\left(\frac{r}{Z}\right)^2}\right]^{5/2} \qquad \rightarrow 3.2$$

And

$$\tau_{rz} = \frac{1}{2}\sigma_R Sin_2\beta$$

$$= \frac{3}{2}\frac{Q}{\pi}\frac{Cos^2\beta Sin\beta}{R^3} = \frac{3}{2}\frac{Q}{\pi}\frac{Z^2 r}{R^5} \qquad \rightarrow 3..3a$$

[redacted]

[redacted]

$$= \frac{3Q}{2\pi}\left[\frac{rz^2}{\left(r^2+z^2\right)^{5/2}}\right]$$

$$= \frac{3Qr}{2\pi z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \rightarrow 3.3$$

It should be emphasized that although both the vertical normal stress and shearing stress are independent of the elastic constants (E and μ) they are very much dependent on the assumptions of linear elasticity.

Equation 3.2 may be written as

$$\sigma_z = K_B \frac{Q}{z^2} \rightarrow 3.4$$

$K_B \Rightarrow$ Boussinesaq influence factor

$$= \frac{3}{2\pi}\left[\frac{1}{\left[1+\frac{r}{2}\right]^2}\right]^{5/2} \rightarrow 3.5$$

The insenties of vertical pressure indirectly below the point load (where r = 0) on its axis of loading given by

$$\sigma_z = \frac{0.4775Q}{Z^2} \rightarrow 3.6$$

Further the horizontal radial stress component σ^r and the displacement u, v and ω in x, y and Z. directions are given by the following expressions.

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3Zr^2}{R^5} - \frac{1-2\varpi}{E(R+Z)}\right] \rightarrow 3.7$$

$$\mu = \frac{Q(1+\mu)}{2\pi E}\left[\frac{xz}{R^3} - \frac{(1-2\mu)x}{R(R+Z)}\right] \rightarrow 3.8$$

$$V = \frac{Q(1+\mu)}{2\pi E}\left[\frac{yz}{R^3} - \frac{(1-2\mu)y}{R(R+Z)}\right] \rightarrow 3.9$$

and

[redacted]

$$W = \frac{Q(1+\mu)}{2\pi E}\left[\frac{z^3}{R^3} + \frac{2(1-\mu)}{R}\right] \qquad \rightarrow 3.10$$

3. Write Notes on Iso-Bars:

An Isobar is a curve or counter connecting all points below the ground surface of equal vertical pressure on a given horizontal plane is the some in all directions at points located at equal radial distances around the axis of loading. The some in a loaded soil mass bounded by on isobar of given vertical pressure. intensity is called a “pressure bulb” the vertical pressure at every point on the surface of pressure bulb is the same.

Suppose on isobar of value $\sigma_z = 0.25.Q$ (or) (25% of Q) per unit area is to be plotted. Then

From Equation 3.4

$$KB = \frac{\sigma_z \times z^2}{Q} = \frac{0.25QZ^2}{Q} = 0.25Z^2$$

A number of numerical values of ‘Z’ are selected and the values of K_B are calculated from (i). Corresponding to these values of KB, r/Z are found from table 3.1 and hence corresponding values r is computed. Thus, we get the coordinates (r_1Z) of a number of points where $\sigma_z = 0.25Q.$ the calculations are better performed in a tabular form shown below.

On any horizontal plane at a depth of σ_Z, Z is the same for the same horizontal distance r on either side of axis of loading this makes the isobar symmetrical about the axis of loading. The depth at which the isobar $\sigma_z = 0.25Q$ crosses the axis of loading is calculated by first finding. KB: when r = 0 thus,

When r = 0, KB = 0.4775

$$\sqrt{\frac{0.4773}{0.25}} = 1.38 \text{ Units.}$$

For any given load system a number of isobars corresponding to various intensities of vertical pressure is drawn. Thus, an “Isobar diagram” in fig, consists of a family of isobars of various intensities.

4. Explain Vertical Pressure Distribution On A Horizontal Plane:

The vertical pressure distribution on any horizontal plane at depth ‘z’ below the ground surface due to a concentrated load is given by

$$\sigma_z = KB\frac{Q}{Z^2}$$

Depth Z is known depth selecting different values of horizontal distance r, KB can be found from table 3.1 and hence σ_z can be computed. Below the load the vertical pressure will be equal to 0.4775 Q/Z^2 and it decreases very rapidly with the increases in the value of r as is evident from table 3.3

From the above table it can be concluded that at a given depth, when horizontal radial distance is equal to twice the depth the vertical pressure due to single concentrated load can be considered negligible.

Fig shows a vertical stress distribution diagram due to a concentrated load at a depth Z. If such a diagram is plotted for unit load (Q = 1) is called the “influence” diagram for point A below the axis. Such a diagram is helpful incomputing the vertical stress σ_z at A due to a number of concentrated loads Q_1, Q_2 …. Q_n etc. Situated at radial distances r_1, r_2 …. r_n from the vertical axis through point A. The vertical stress is then given by

$$\sigma_z = \sum Q.O$$

$$= Q_1O_1 + Q_2O_2 + ... + Q_nO_n \qquad \rightarrow 3.11$$

Where o, 01, 02…………………on are the ordinates of the influence diagram plotted for σ_Z at A.

Te influence diagram can be used to find σ_Z at any point on a horizontal plane, by or lending the diagram on the plane in such a way that vertical axis through that point coincides with the maximum ordinate (O) of the influence diagram when once this is done the ordinates 01, 02, …. o_n due to any given system of loads can be found and σ_z can be computed form equation 3.11

5. Explain The Vertical Pressure Distribution On Vertical Line:

From Equation 3.4, It is clear that σ_z also decreases with increase in the depth Z on any vertical line distant r from the axis of the load, the variation of σ_z can be plotted from the relation.

$$\sigma_z = KB\frac{Q}{z^2}$$

In the above expression the radial distance re associated with KB is constant. Hence various values of Z and r/Z can be selected and KB can be found. Then σ_z can be computed which will be proportional of KB/Z^2. A table can be prepared as under.

Fig shows the vertical stress distribution on a vertical line at distance r from the axis of loading the vertical stress first increases attains a maximum values and then decreases. It can be shown that the maximum value of σ_z on a vertical line is obtain at the point of intersection of the vertical plane with a radial line at $\beta = 39^o\ 15'$ through the point load as shown in fig .. The corresponding value of $\frac{r}{z} = 0.817$

[redacted]

$$Z = \frac{r}{0.817} = \frac{1}{0.817} = 1.225$$

KB = 0.1332

Hence

$$\sigma_{z\max} = \frac{0.1332Q}{(1.225)^2} = 0.0888Q$$

6.Find the intensity of vertical pressure and horizontal shear stress at point 4m directly below a 20 KN point load acting at a horizontal ground surface what will be vertical pressure and shear stress at a point 2m horizontal away from the axis of loading but at the same depth of 4m.

Solution

Given

r = 0

Q = 20 K N

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{3\times 20}{2\times\pi\times(4)^2} = 0.597KN/m^2$$

From equation 3.3

$$\tau_{rz} = \frac{3Q}{2\pi}\frac{rz^2}{\left(r^2+z^2\right)^{5/2}} = \frac{3Qr}{2\pi 23}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^{5/2}$$

= 0 (sine r = 0)

Alternatively, from table 3.1

$$KB\left(for\ \frac{r}{z} = 0\right) = 0.4775$$

$$\sigma_z = KB\frac{Q}{z^2} = \frac{0.4775}{(4)^2}\times 20 = 0.597\,KN/m^2$$

r = 2 m: z = 4 m ; $\frac{r}{z} = 0.5$

$$\sigma_z = \frac{3\times 20}{2\pi\times 42}\left[\frac{1}{1+(0.5)^2}\right]^{5/2} = 0.342\,KN/m^2$$

[redacted]

$$\tau_{rz} = \frac{3\times 2\times 20}{2\pi \times 42}\left[\frac{1}{1+(0.5)^2}\right]^{5/2} = 0.171 KN/m^2$$

Alternatively,

$$KB\left(for\,\frac{r}{z} = 0.5\right) = 0.2733$$

$$\sigma_z = KB\frac{Q}{z^2} = \frac{0.2733}{(4)^2}\times 20 = 0.342 KN/m^2$$

$$\tau_{rz} = \sigma_z.\frac{r}{z} = 0.342\times\frac{0.5}{2} = 0.171 KN/m^2$$

7.Prove the maximum vertical stress n a vertical line at a constant radial distance r from the axis of a vertical load is induced at the pint of intersection of the vertical line with a radial line at β = 39º 15’ from the point of application of concentrated load. What will be the value of shear stress at the hence or otherwise find the maximum vertical stress on a line situated at r = 2 m from the axis of a concentrates load of value 20 KN.

Solution

We have

$$\sigma_z = \frac{3Q}{2\pi}.\frac{Z^3}{\left(x^2+r^2\right)^{5/2}} = \frac{3Q}{2\pi Z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

For the maximum value of σ_z (where r is constant) differentiate equation 3.2 with respect to ‘Z’ and equate it to zero.

$$\frac{d\sigma_z}{dz} = \frac{3Q}{2\pi}\left[\frac{3z^2\left(r^2+z^2\right)^{5/2} - z^3\times\frac{5}{2}\left(z^2+r^2\right)^{3/2}2z}{\left(Z^2+r^2\right)^{5/2}}\right] = 0$$

$$3z^2\left(r^2+z^2\right) - 5z^4 = 0$$

from which $Z = \left(\sqrt{\frac{3}{3}}\right)r = 1.225r$ →3.2

$$\frac{r}{z} = \sqrt{\frac{2}{3}} = \frac{1}{1225} = 0.817 = \tan\beta$$

Substituting the value of $\frac{r}{z} = \sqrt{\frac{2}{3}}$ and $\sqrt{\frac{3}{2}}r$ in equation →3.12 we get.

[redacted]

$$(\sigma_z)_{\max} = \frac{3Q}{2\pi}\left[\frac{1}{\left(\sqrt{\frac{3}{2}}\right)^2}\left[\frac{1}{1+\frac{2}{3}}\right]^{5/2} = \frac{Q}{\pi r^2}\times\frac{1}{\left(1+\frac{2}{3}\right)^{5/2}}\right] = 0.0888\frac{Q}{r^2}$$ 13.13

$$\tau_{rz} = \frac{3Q}{2\pi}\cdot\frac{rz^2}{\left(r^2+z^2\right)^{5/2}} = \frac{3Qr}{2\pi Z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\sigma_{z\max}\frac{r}{z} = \left(0.0888\frac{Q}{r^2}\right)\times 0.817 = 0.0725\frac{Q}{r^2}$$

Where r = 2 m and Q = 20 KN, $\sigma_{z\max} = 0.0888\times\frac{20}{4} = 0.444 KN/m^2$

z = 1.225 r = 2.45 m

$$\tau_{rz} = 0.0725\frac{Q}{r^2} = \frac{0.0725\times 20}{4} = 0.362 KN/m^2$$

8. Explain the Vertical Pressure under a uniformly loaded circular Area

The Bussinesq equation for the vertical stress due to signal concentrated load can now be extended to find the vertical pressure loaded circular area fig Shows a uniformly loaded circular area of radius a and load intensity q per unit area. Assume the soil as an elastic isotropic semi – infinite mass.

Consider an elementary ring of radius r and width δr on the loaded area. If the elementary ring is further divided in small parts each of area δA the load on each elementary area will be of δA. This load may be considered as a point load. Hence the vertical pressure at point P, situated depth z on the vertical axis through the centre of the area is evidently given by equation 3.2

$$\delta\sigma_z = \frac{3(q.8A)}{2\pi}\cdot\frac{Z^3}{\left(r^2+z^2\right)^{5/2}}$$

Integrating over the entire ring of radius r_1 the vertical stress $\Delta\sigma z$ is given by

$$\Delta\sigma z = \frac{3q}{2\pi}\left(\sum\delta A\right)\frac{Z^3}{\left(r^2+z^2\right)^{5/2}} = \frac{3q(2\pi r\delta r)}{2\pi},\frac{Z^3}{\left(r^2+Z^2\right)^{5/2}}$$

[redacted]

[redacted]

$$= 3qr\delta r \frac{Z^3}{\left(r^2 + Z^2\right)^{5/2}}$$

the total vertical pressure σ_z due the entire loaded area is given by integrating the above expression between the limits.

r = 0 and r = a

$$\sigma_z = 3qz^3 \int_0^a \frac{rdr}{\left(r^2 + z^2\right)^{5/2}}$$

put $r^2 + Z^2 = n^2$; so that r dr = n dn.

Limits = when r = 0 and n = z

When r = a and $n = \left(a^2 + Z^2\right)^{1/2}$

[redacted]

$$\therefore \quad \sigma_z = 3qz^3 \int \left(a^2 + z^2\right)^{1/2} \frac{dn}{n^4}$$

$$= qz^3 \left[\frac{1}{z^3} - \frac{1}{\left(a^2 + z^2\right)^{3/2}} \right]$$

$$= \sigma_z = q \left[1 - \frac{1}{\left(1 + \frac{a}{z}\right)^2} \right]^{3/2} \qquad \rightarrow 3.15$$

$$\sigma_z = K_3 . q \qquad \rightarrow 3.16$$

where K_B = Boussineqa influence factor for uniformly distributed circular load.

$$= 1 - \left[\frac{1}{1 + \left(\frac{a}{z}\right)^2} \right]^{3/2} \qquad \rightarrow 3.16.a$$

Table 3.5 given the value of the influence factors for various values of q/z. The vertical pressure at a given depth on the vertical axis through the centre of the circular loaded area can be found by multiplying the influence factor by the load intensity q. For the vertical pressure at any point not situated under the centre of the circular load,

If θ is the angle which the line joining the point P‘ makes with the outer edge of the loading equation 3.15 reduces to

$$\sigma_z = q\left[1 - Cos^3\theta\right]$$

fig shows a family of isobars under a uniform loaded circular area first presented by jurgenson (1934).

With the help of this diagram the vertical pressure of various points below a circular load area can be conveniently determined.

9. Explain the Vertical Pressure Due To a Line Load.

Let us consider an infinitely long line load of intensity ‘q’ per unit length, acting on the surface of a semi-infinite elastic medium. Let the y –axis be directed along eh direction of the line load, as shown fig.. Let us fine the expression for the vertical stress at any point ‘P’ having co-ordinates (x, y, z)

The radial distance of point $p = r = \left(x^2 + y^2\right)^{1/2}$ the polar distance of point P

$$= R = \left(r^2 + z^2\right)^{1/2}, \left(x^2 + y^2 + z^2\right)^{1/2}$$

Consider a small length δy along the line load the elementary load n this length will be equal to q’. δy which can be considered to be a concentrated load hence the vertical stress $\Delta\sigma_z$ due this elementary load is given by,

$$\Delta\sigma_z = \frac{3(q'\delta y)z^3}{2\pi R^5} = \frac{3q'\delta y z^3}{2\pi\left(x^2 + y^2 + z^2\right)^{5/2}}$$

$$= \int_{-\infty}^{\infty} \frac{3q' Z^3 dy}{2\pi\left(x^2 + y^2 + Z^2\right)^{5/2}} = 2\int_{0}^{\infty} \frac{3q' Z^3 dy}{2\pi\left(x^2 + y^2 + Z^2\right)^{5/2}}$$

$$\sigma_z = \frac{2q' Z^2}{\pi\left(x^2 + Z^2\right)^4} = \frac{2q^1}{\pi z}\left[\frac{1}{\left[1 + \left(\frac{x}{z}\right)^2\right]^2}\right] \qquad \rightarrow \quad 3.18$$

In the above expression x and z are constants for a given position of a point P and the only variable is y. Also , x is the horizontal distance of point ‘P’ from the line load, in direction perpendicular to the line load when the point ‘P’ is situated vertically below the line load, at a depth Z, e have x = 0 and hence the vertical stress is given by

$$\sigma_z = \frac{2q^1}{\pi z} \qquad \rightarrow 3.19$$

[redacted]

10. Explain the Vertical Pressure under Strip Load.

Fig shows an infinite strip of width B, loaded with uniformly distributed load intensity of per unit area. Let us find the vertical pressure at a point P situated below a depth Z on a vertical axis passing through the centre of the strip.

Consider a strip load of width dx at distance x from the centre. Te elementary strip of width dx will be q.dx. The vertical pressure at P due to this elementary line load is given by equation 3.18.

$$\Delta\sigma z = \frac{2qdx}{\pi z} \cdot \frac{1}{\left[1+\left(\frac{x}{z}\right)^2\right]^2}$$

Total vertical pressure due to the whole strip load is given by.

$$\sigma z = \frac{2q}{\pi z}\int_{-B/2}^{B/2} \frac{dx}{\left[1+\left(\frac{x}{z}\right)^2\right]^2} = \frac{4P}{\pi Z}\int_{0}^{B/2} \frac{dx}{\left[1+\left(\frac{x}{2}\right)^2\right]^2}$$

put $\frac{x}{z} = \tan\beta,$ $\therefore\ dz = Z\sec^2\beta$

$$\therefore\ \sigma_z = \frac{4q}{\pi}\int_{0}^{\theta/2} \frac{\sec^2\beta . d\beta}{\left(1+\tan^2\beta\right)^2}$$

$$= \frac{4q}{\pi}\int_{0}^{\theta/2} Cos^2\beta d\beta = \frac{q}{\pi}\left(\theta + Sin\theta\right). \rightarrow (3.20)$$

Table 3.6 gives vertical pressure at different depths below the centre of a uniform load of intensity q and width B.

11. Explain The Vertical Pressure Under A Uniformly Loaded Rectangular Area:

Let us take the case of a rectangular load area of length 2 a and width 2b, and let the reference axes pass through the centre of the area as shown in fig. Let the point ‘P’ where vertical stress has to be fond have to co-ordinates (x, y, z)

Consider an elementary area $\delta A = \delta\xi\,\delta\eta$ and let the dx and y co-ordinates of the centre of this elementary area be ξ and η respectively. Evident l , the x and y co-ordinates of point P with respect to the elementary area will be $(x-\xi)$ and (y - η) respectively. Hence the polar distance R between the elementary load and point P is given by

[redacted]

[redacted]

$$R=\left[(x-\xi)^2+(y-\eta)^n+Z^2\right]^{1/2}$$

The vertical stress $\Delta\sigma_z$ at P due to this elementary load $(\delta\xi,\ \delta n).q$ is given by Equation 3.2

$$\Delta\sigma_z=\frac{3}{2\pi}(\delta\xi\delta n\ q)\frac{r^3}{R^5}$$

Hence the vertical stress at P, due to the entire loaded area is given by

$$\sigma_z=\frac{3qz^3}{2\pi}\int_{-a}^{a}\int_{-b}^{b}\frac{d\xi d\eta}{\left[(x-\xi)^2+(h-\eta)^2+Z^2\right]^{5/2}}$$

Florin (1959,61) obtained the above integral However the integral is for too lengthily to be of partical value . A more practical case is the vertical stress (σZ) under the center of the rectangle (x = y = 0)

$$(\sigma_z)_0=\frac{3qz^3}{2\pi}\int_{-a}^{a}\int_{-b}^{b}\frac{d\xi d\eta}{\left(\xi^2+\eta^2+Z^2\right)^{5/2}}$$

$$(\sigma_z)_0=\frac{2q}{\pi}\left[\frac{ab_2\left(a^2+b^2+2z^2\right)}{(a+z)^2(b+z)^2\sqrt{a^2+b^2-Z^2}}+Sin^{-1}\frac{ab}{\sqrt{a^2+Z^2}\sqrt{b^2+Z^2}}\right.\quad\rightarrow 3.21$$

The above expression can now be utilized to find the vertical stress under the corner of a rectangular area of size a, b from principle of super position the vertical stress under the corner, of eh rectangle of size a, b will be one quarter of the above expression.

$$(\sigma_Z)_c=\frac{q}{2\pi}\left[\frac{m^1n^1}{\sqrt{1+m^{12}+n^{12}}}\cdot\frac{1+m^{12}+2n^{12}}{\left(1+n^{12}\right)\times\left(m^{12}+n^{12}\right)}+Sin^{-1}\frac{m^1}{\sqrt{m^{12}+n^{12}\sqrt{\left(1+n^{12}\right)}}}\right.$$

$$\rightarrow 3.22$$

$$(\sigma_z)_c=qKs\ \rightarrow 3.23$$

Where Ks = Steinbrenner (1936) influence factor given by curves of fig

A more common from of the vertical stress under the corner of a rectangular area of size a, b is as follows.

$$(\sigma_Z)_c=\frac{q}{4\pi}\left[\frac{2mn\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1}+\tan^{-1}\frac{2mn\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2-m^2n^2+1}\right.$$

[redacted]

[redacted]

$\rightarrow$ 3.23

$$(\sigma_z)_c = Kq \rightarrow 3.24$$

$$\left(where\ m = \frac{a}{z}, n = \frac{b}{z} \right)$$

Where, K = influence factor. Table 3.7 gives the values of influence factor K. In the above formulate a and b or m and n are inter changeable. The above form of solution is after new mark (1935)

Equation 3.24 can also be utilized for vertical stress at a point 'P' not situated at the corner of the rectangle, but below some other point 'A' either inside or outside the rectangle as shown in fig 3.11 and 3.12 when the point A is inside the rectangle the vertical stress at point P, vertical below A at depth Z is given by

$$\sigma_z = q(K_1 + K_2 + K_3 + K_4) \rightarrow 3.25$$

where K_1, K_2, K_3, K_4 are the influence factors for the four rectangles 1,2,3 and 4 similarly if the point A is outside the loaded rectangle, construct the four rectangles as shown in fig 3.12. The shaded area in the loaded rectangle ma be considered to be the algebraic sum of the four rectangles each with the corner at A;

Area abcd = Ab_1Cd_1 + $Ab_1b\,a_2$ - $Aa_1\,dd_1$ + $A\,a_1\,a\,a_2$

$\therefore\ \sigma_z$ at A = q (K_1 - K_2 - K_3 + K_4) $\rightarrow$ 3.26

Where

K_1	=	Influence factor for area A b_1 cd_1
K_2	=	Influence factor for area A b_1 ba_2
K_3	=	Influence factor for area A a_1 dd_1
K_4	=	Influence factor for area A a_1 aa_2

12. a. Explain the Equivalent Point Load Method

This is an approximate method of calculating the vertical stress at any point due to any loaded area. The entire area is divided into a number of small area units and the total distributed load over a unit area is replaced by a point load of the same magnitude acting at the centroid of the area unit.

Thus, the distributed load over the whole area is replaced by a number of point loads situated at the centroids of the various area units. The influence factors for each of these load positions can be found with respect to the point P where σ_z is to be determined. The vertical stress is then given by

$$\sigma_z = \frac{1}{Z^2}\left[Q_1K_{B1} + Q_2K_{B2}....Q_nK_{Bn}\right] \rightarrow 3.27$$

If all the points loads are of equal magnitude Q^1

[redacted]

$$\sigma_z = \frac{Q^1}{Z^2}\sum K_B \qquad \rightarrow 3.28$$

Where,

$\sum K_B =$ Sum of the individual influence factor for the various area units.

The accuracy of the result will depend upon the size of the area unit chosen. If the length of the side of the small area unit is less than one-third of the depth at which vertical pressure is required, the error involved in the result is within 3 percent.

12..b Explain the Newmark's influence chart

A more accurate method of determining the vertical stress at any point under a uniformly loaded area of any shape is with the help of influence chart or influence diagram original suggested by Newmark (1942). A chart, consisting of number of circles and radiating lines, is so prepared that the influence of each area unit (formed in the shape of a sector between two concentric circles and two adjacent, radial lines) is the same at the centre of the circles, i.e., each area unit causes the equal vertical stress at the centre of the diagram.

Let a uniformly loaded circular area of radius r_1 cm be divided into 20 sectors (area units) as shown in fig. 13.14. If q is the intensity of loading, and σ_z is the vertical pressure at a depth Z below the centre of the area, each unit such as OA, B, exerts a pressure equal to $\frac{\sigma_z}{z\sigma}$ at the centre.

Hence, from equation 3.15

$$\frac{\sigma_z}{20} = \frac{q}{20}\left[1-\left\{\frac{1}{1+\left(\frac{r_1}{Z}\right)^2}\right\}^{3/2}\right] = i_f q \qquad \rightarrow 3.29$$

where i_f = influence value

[redacted]

$$= \frac{1}{20}\left[\left\{1 - \frac{1}{1+\left(\frac{r_1}{Z}\right)^2}\right\}^{3/2}\right]$$

If i_f be made equal to an arbitrarily fixed value say 0.005,

We have

$$\frac{q}{20}\left[1 - \left\{\frac{1}{1-\left(\frac{r_1}{Z}\right)^2}\right\}^{3/2}\right] = 0.005q \qquad \rightarrow 13.30$$

Selecting the value of Z = 5 cm (say), the value of r_1 solved from equation 13.30 comes out be 1.35 cm. Hence if a circle is drawn with radius r_1 = 1.35 cm and divided into 20 equal area units, each area unit will exert a pressure equal to 0.005 q intensity at a depth of 5cm.

Let the radious of second concentric circle be equal to r_2 cm. By extending the twenty radial lines, the space between the two concentric circles is again divided into 20 equal area units; A_1 A_2 B_2 B_1 is one such area unit. The vertical pressure at the centre, due to each of these area nits is to be intensity 0.005 q. Therefore, the total pressure due to area units OA, B_1 and A1A2B2B1 at depth z = 5 cam below the centre is 2 x 0.005 q. Hence from equation 3.15

Vertical pressure due to OA_2 B_2

$$= \frac{q}{20}\left[1 - \left\{\frac{1}{1+\left(\frac{r_2}{Z}\right)^2}\right\}^{3/2}\right] = 2 \times 0.005q$$

Substituting z = 5 cm, we get r_2 = 2.00 cm from the above relation. Similarly, the radii of 3^{rd} , 4^{th} , 5^{th} 6^{th} 7^{th} 8^{th} 9^{th} circles can be calculated as tabulated in table 13.8. The radius of 10^{th} circle is given b the following governing equation:

[redacted]

[redacted]

$$\frac{q}{20}\left[1-\left\{\frac{1}{1+\left(\frac{r_{10}}{Z}\right)^2}\right\}^{3/2}\right]=10\times 0.005q=\frac{q}{20}$$

From the above r_{10} = infinity.

Fig shows the influence chart drawn on the basis of table 3.8

To use the chart for determining the vertical stress at any point under the loaded area, the plan of the loaded area is first drawn on a tracing paper to such a scale that the length ABH (= 5 cm) drawn on the chart represents the depth to the point at which pressure is required. For example, if the pressure is to be found at a depth of 5m, the scale of plan will be 5cm = 5m , or 1cm = 1m. The plane of the loaded area is then 30 placed over the chart that the point below which pressure is required coincides with the centre of the chart. The point below which pressure is required may lie within or outside the loaded area. The total number of area units (including the fractions covered by the plan of the loaded area is counted. The vertical pressure is then calculated from the relation)

$$\sigma_A = 0.005q\times N_A \qquad \rightarrow 3.31$$

where, N_A = number of area units under the loaded area.

13. A rectangular area 2m x 4m carries a uniform load of 80 KN/m² at the ground surface find the vertical pressures at 5m below the centre and corner of the loaded area..

Solution

(a) For the point under the centre of the area , there will be influence of four rectangles of size 1m x 2m having a common corner at the centre of the loaded rectangle.

a = 1 m, b = 2 m.

$$m=\frac{a}{z}=\frac{1}{5}=0.2;n=\frac{b}{z}=\frac{2}{5}=0.4$$

K_{B1} (for one quadrant) = 0.0328

$$\sigma_z = 4qK_{B1}4\times 80\times 0.0328=10.5KN/m^2$$

(b) for the point under the corner of rectangle;

a = 2 m; b = 4 m

$$m=\frac{2}{5}=0.4; \quad n=\frac{4}{5}=0.8$$

K_B = 0.0931

$$\sigma_z = qK_B = 80\times 0.0931 = 7.45KN/m^2$$

14. A rectangular area 2m x 4m carries a uniform load of 80 KN/m² at the ground surface find the vertical pressures at 5m below the centre and corner of the loaded area.. **Solve the problem by the equivalent load method.**

Solution:

[redacted]

[redacted]

Divide loaded area into four equal rectangular of size 1m x 2m . Each area will represent a point load Q^1 = 1 x 2 x 80 = 160 KN acting at its centroid

(a) for the point under the centre

Te influence of each area unit will be equal

$$r^1 = \sqrt{1+(0.5)^2} = 1.117$$

$$\frac{r^1}{Z} = \frac{1.117}{5} = 0.223$$

K_B = 0.4247

$$\sigma_z = \frac{Q^1}{Z^2}\sum K_B$$

$$= \frac{160 \times 4 \times 0.4247}{5 \times 5} = 10.85 KN/m^2$$

By exact method = 10.5 KN/m^2 (for 15 problem)

% error $= \dfrac{10.85 - 10.5}{10.5} = 3.3\%$

(b) For the point under corner B

The influence of each area unit will be different. Let r_1, $r_2 r_3$, r_4 be the radial distance of centroids of each unit from B.

The corresponding value of $\dfrac{r}{z}$ and K_B are as under:

Area unit	r	$\dfrac{r}{z}$	K_B
1	1.117	0.223	0.4247
2	3.040	0.608	0.2174
3	3.360	0.672	0.1880
4	1.800	0.360	0.3521

$$\sum K_B = 1.1822$$

$$\sigma_z = \frac{Q^1}{Z^2}\sum K_B = \frac{160 \times 1.1822}{25} = 7.56 KN/m^2$$

But by exact method

$$\sigma_z = 7.45 KN/m^2$$

% error $= \dfrac{7.56 - 7.45}{7.45} = 1.56\%$

[redacted]

15. A rectangular area 2m x 4m carries a uniform load of 80 KN/m^2 at the ground surface find the vertical pressures at 5m below the centre and corner of the loaded area. Using Newmark's influence chart.

Solution:

Z = 5 m

Hence the scale of the plan will be

AB (= 5 cm) = 5m   or 1 cm = 1m

(a) The plan of the rectangular area is drawn to the scale of 1cm = 1m, and oriented on the chart in such a way that is centroid is over the centre of the diagram.

Number of area unit under the rectangle = N_A = 25.5 Units.

σ_z under the centre of area = 0.005 x qN_A

= 0.005 x 80 x 25.5 = 10.2 KN / m^2

(b) The plan of the rectangular area is then oriented in such a way that is of its corner is above the centre of chart. Then

N_A = 18.5 Units.

σ_z under corner of area = 0.005 x 80 x 18.5

= 7.4 KN/m^2

16. Explain the Westergaard's Analysis?

Westergaard (1938) also solved the problem of pressure distribution in soil under point load, assuming the soil to be an elastic medium of semi-infinite extent but containing numerous closely spaced, horizontal Sheets of negligible thickness of an infinite rigid material which permits only downward deformation on the mass as a whole without allowing it to undergo any lateral strain.

Te assumption of no lateral displacement implies that

$$U = V = \in x = \in y = \gamma_{xy} = 0$$

And volume dilatation = $\in = \in z = \dfrac{\gamma\ \omega}{\gamma\ z}$

Where u, v and ω are displacement in x, y and z directions. The stresses on a horizontal plane, which are now simply function of vertical displacement are given by

$$\sigma_z = \frac{2(1-\mu)G}{(1-2\mu)}\frac{\gamma\omega}{\gamma x} \quad \rightarrow 3.31.a$$

where, G = Shear modulus $= \dfrac{E}{2(1+\mu)}$

$$\tau_{xZ} = G\left(\frac{\gamma\omega}{\gamma x}\right) \quad \rightarrow 3.31.b$$

and $$\tau_{xy} = G\left(\frac{\gamma\omega}{\gamma y}\right) \qquad \rightarrow 3.31.c$$

Substituting these into equilibrium equations in the vertical direction. We get

$$\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} + \frac{\partial^2\omega}{\partial z^2} = \nabla^2\omega = 0 \qquad \rightarrow 3.32$$

where , $Z = \eta Z,$ and $$\eta = \sqrt{\frac{1-2\mu}{2(1-\mu)}} \qquad \rightarrow 3.33$$

For the point load Q applied at the origin of the co-ordinates, Westergaard obtained,

$$W = \eta\frac{Q}{2\pi RG} \qquad \rightarrow 3.34$$

where $R^2 = r^2 + Z^2 = x^2 + y^2 + \eta^2 Z^2$.

Substituting in equation 3.21 a, we get

$$\sigma_Z = \frac{Qz}{2\pi R^3} = \frac{Q}{2\pi\eta^2 Z^2\left[1+\left(\frac{r}{\eta Z}\right)^2\right]^{3/2}} \qquad \rightarrow 3.35$$

The value of μ varies from 0 to 0.5 for elastic materials, For a case of large lateral restraint, the lateral strain is very small and μ maybe assumed as zero, $\sum q$ then reduces to.

$$\sigma_Z = \frac{1}{\pi\left[1+2\left(\frac{r}{Z}\right)^2\right]^{3/2}}\frac{Q}{Z^2} = Kw\frac{Q}{Z^2} \qquad \rightarrow 3.36$$

where Kw = Westergaard influence factor

$$= \frac{1}{\pi\left[1+2\left(\frac{r}{z}\right)^2\right]^{3/2}} \qquad \rightarrow 3.37$$

17. Explain the Contact Pressure?

Contact pressure is defined as the vertical pressure acting at the surface of contacts between the base of a footing and the underlying soil mass.

To simplify design, the computation of the bending moments etc. in the footings is commonly based on the assumption that the footing rest on a uniformly spaced bed of sprig so that the distribution of contact pressure is uniform. The actual contact pressure distribution, however, depends upon the flexural rigidity of the footing and the elastic properties of the subgrade.

If the footing in flexible, the distribution of contact pressure is uniform irrespective of the type of the subgrade or under–soil material. If the footing is perfectly rigid, the contact pressure distribution depends upon the type of the subgrade. Fig shows the pressure distribution under rigid footing resting over (a) real, elastic material (such as saturated clay) and (b) cohesion-less sand and (c) soil having intermediate characteristics.

In the case of a real elastic material, theoretical intensity of contact pressure at the centre is q/z and infinite at the outer edges. However, local yielding causes redistribution of pressure making it finite at eh edges. When the loading approaches a value sufficient to cause failure of soil, the contact pressure distribution may probably be very nearly uniform.

In the case of sand, no resistance to deformation is offered at the outer edges of the footing, making the contact pressure zero there. The pressure distribution is parabolic with maximum value at the centre, through it tends to become more uniform with increasing footing width.

When a footing is neither perfectly flexible, nor perfectly rigid and the underlying soil possesses both cohesion and friction, the contact pressure lies between the extreme conditions for uniform and nonuniform distribution for flexible and rigid footings.

\

18.a. Explain The One Dimensional Consolidation:

When a compressive local is applied to soil mass, a decrease in its volume takes place. The decrease in the volume of soil mass under stress is knows as compression.

Every process involving a decrease in the water content of a saturated soil without replacement of the water by air is a called process of consolidation.

The compressibility of clays may also be caused by three factors

i. To expulsion of double layer water from between the grains
ii. Slipping of the particles to new positions of greater density
iii. Bending of particles as elastic sheet. The clay buy very small.

18.b.Explain The Consolidation Process : Spring analogy

Let the length of the spring be Z_0 under a pressure of 10 unit it 12 units pressure are added to it top. The spring will be compressed immediately to a length Z_1 A further application of load will result in further decrease in the length of the spring with elastic limit load diffusion curve may be assumed to be straight. If this spring and piston is placed in a cylinder containing water will be tree to stress since the whole load is carried by the spring alone. It of stress sine the whole load carried by the spring alone. If the pressure on the piston is increased to 12 units. Valve is closed the spring cannot design since water is incompressible. Hence the additional total pressure σ^1.

[redacted]

$$\sigma = \sigma^1 + \bar{u}$$

Now the value be opened slightly so that some water escapes and then valve is closed. Due to escape of some water the position moves down,, the spring is compressed and hence some pressure out of pressure of 2 units entirely burns by water is now transferred to the spring.

$$12 = (10 + \Delta\sigma^1) + (2 - \Delta\sigma^1)$$

ere $\Delta\sigma^1$ is the transfer of pressure from water to the spring corresponding to a given amount of expulsion of water. If the valve is fully opened. Sufficient water will escape till the length of spring is reduced to a height of Z_1

$$12 = 12 + 0$$

$$\sigma = \sigma^1 + \bar{u}$$

Thus we see that when there is a pressure increment, the whole of pressure is first taken by water. As the water escape out of the system. The load transfer takes place form water the spring till the spring is deformed by the full amount corresponding to the applied stress increment.

This analogy can be applied stress increment. This analogy can be applied consolidation process of a soil mass consisting of soil water system. The grain structure represents the spring while the voids filled with water represent the cylinder.

The pressure that builds up in pore water due to load increment on the soil is termed excess pore pressure or excess hydrostatic pressure or hydrodynamic pressure $\bar{u}$, because it is excess of the initial pressure in water under to clear out of the voids. No more water escapes from the voids and condition of equilibrium is attained. The delay caused in consolidation by the slow drainage of water out of a saturated soil mass is called hydrodynamic lag.

19. Expalin the Consolidation of Laterally Confined Soil?

If a remoulded soil is laterally confined in a consolidometer consisting of a metal ring and porous stones are placed both at its top and bottom faces the compression or consolidation sample takes places under a vertical pressure applied on the top of porous stones.

The porous stones provide free drainage of water and air from or into the soil sample. Under a given applied pressure a final settlement and equilibrium voids ratio is attained after certain time. At the equilibrium stage the applied pressure naturally becomes the effective pressure σ^1 on the soil. The pressure can be increased end a new equilibrium voids ratio is attained. Thus a relationship can be obtained between effective pressure σ^1 and the equilibrium voids ratio (e) curve $\sigma^1 = 300 KN/m^2$ pressure is completely removal compression mainly due to some irreversible orientation undergone by the sail particles under compression of the soil is again put under compression, a recompression curve such CD is voids ratio at D being always less than that at B at some pressure 300 KN/m^2 pressure increments. DE B portion curve represents the compression of a soil which has not been subcided it the pressure.

$$\ell = \ell_0 - C_c \log_{10} \frac{\ell^1}{\ell^1 0}$$

where,

[redacted]

$\ell_0 =$ initial voids ration corresponding initial.

e = , Voids ration at increased pressure σ^1

C_c = Compression index.

The compression index represents the slope of the linear portion of pressure voids.

$$C_c = \frac{e_0 - e}{\log 10 \dfrac{\sigma^1}{\sigma_0{}^1}} = \frac{\Delta e}{\Delta \log_{10} \sigma^1}$$

$$\Delta_e = C_c \log_{10} \frac{\sigma_0{}^1 + \Delta\sigma^1}{\sigma_0{}^1}$$

$$e_0 = e^t C_s \log_{10} \frac{\sigma^1}{\sigma_0{}^1}$$

C_s = expansion or succeeding index.

Consolidation tests on a number of class

$$C_c = 0.007(\omega_L - 10\%)$$

For a ordinary clay of medium to low sensitives.

$$C_c = 0.009(\omega_L - 10\%)$$

Coefficient of compressibility a $_v$

$$a_v = \frac{-\Delta_e}{\Delta\sigma^1} = \frac{e_0 - e}{\sigma^1 - \sigma_0^1}$$

For a given difference in pressure the value of coefficient of compressibility

Co efficient of volume change

$$mv = \frac{-\Delta e}{1 + e_0} \frac{1}{\Delta\sigma^1}$$

[redacted]

$$\frac{-\Delta_e}{\Delta\sigma^1} = av$$

$$mv = \frac{av}{1+e_0}$$

when the soil is laterally confined, the changes in the volume is proportional to change in the thickness ΔH and initial volume is proportional initial thickness H_0.

$$mv = -\frac{\Delta H}{H_0}\frac{1}{\Delta\sigma^1}$$

$$\Delta H = -mvH_0\Delta\sigma^1$$

20. Explain the Terzaghi's Theory of One Dimensional Consolidation?

Te theoretical concept of the consolidation process was developed by Terzaghi in the development of the mathematical statement of the consolidation process. The following

1. soil homogenous and fully saturated
2. Deformation of the soil is due entirely to change in volume
3. Darcy's law for the velocity of flow of water through soil is perfectly valid.
4. Coefficient of permeability is constant during consolidation
5. Load is applied deformation occurs only in direction
6. The change in thickness of the layer during consolidation is insignificant.

Figure shows,a Clay layer of thickness H between two layer of sand which serves as drainage face. When the layer is subjected to a pressure increment $\Delta\sigma_1$ excess hydrostic pressure application whole of the consolidating pressure $\Delta\sigma$ is carried by the pore water so that hydrostic pressure to is a_0 equal $\Delta\sigma$.

$$h = \frac{\overline{u}}{\gamma\omega} \qquad \text{(i)}$$

$$i = \frac{\partial h}{\partial z} = \frac{1}{\gamma\omega}\frac{\partial\overline{u}}{\partial z} \qquad \text{(ii)}$$

$$V = Ki = \frac{K}{\gamma\omega}\frac{\partial\overline{u}}{\partial z} \qquad \text{(iii)}$$

Change of velocity along depth layer.

$$\frac{\partial v}{\partial z} = \frac{K}{\gamma\omega}\frac{\partial^2\overline{a}}{\partial z^2} \qquad \text{(iv)}$$

The velocity of the exist will be equal to $V + \frac{\partial v}{\partial z}dx$

[redacted]

The quantity of water leaving soil elements

$$= \left(V + \frac{\partial v}{\partial z} dz \right) \qquad \text{(v)}$$

$$\Delta q = \frac{\partial v}{\partial z} dxdydz \qquad \text{(vi)}$$

$$\Delta v = -mvV_0 \Delta \sigma^1 \qquad \text{(vii)}$$

$$\frac{\partial (\Delta v)}{\partial t} = -mvdxdydz \frac{\partial (\Delta \sigma^1)}{\partial t} \qquad \text{(viii)}$$

$$\frac{\partial v}{\partial z} = -mv \frac{\partial (\Delta \sigma^1)}{\partial t} \qquad \text{(viii)}$$

$$\Delta \sigma = \Delta \sigma^1 + \bar{u}$$

$$\frac{\partial (\Delta \sigma^1)}{\partial t} = \frac{-\partial \bar{u}}{\partial t} \qquad \text{(ix)}$$

(i) and (ix)

$$\frac{\partial v}{\partial z} = mv \frac{\partial \bar{u}}{\partial t}$$

(iv) and (x)

$$\frac{\partial \bar{u}}{\partial t} = \frac{K}{mv\gamma\omega} \frac{\partial^2 \bar{u}}{\partial z^2}$$

$$\frac{\partial \bar{u}}{\partial t} = Cv \frac{\partial^2 \bar{u}}{\partial z^2}$$

Cv = Coefficient of consolidation

$$= \frac{K}{mv\gamma\omega}$$

$$= \frac{K(1+e_0)}{a_v \gamma\omega}$$

Basic differential equation of consolidation which related the rates of changes of excess hydrostic pressure is the rate of expulsion of excess proves loaded form a unit volume of soil during the same time interval. The term coefficient of consolidation e_v used in the equation is adopted to indicate the combined effects of permeability and compressibility of soil on the rates of volume change Cv on cm^2 / sec.

21. Explain The Solution Of The Consolidation Equation:

The solution of the differential equation of consolidation is obtained by means of the Fourier series

i. At t = 0 at distance z, $\bar{u} = u_0 = \Delta\sigma$

ii. At t = ∞ $z, \overline{u_0} = 0$

iii. At t = t at z = 0, $\bar{u} = 0$

at Z = H, $\bar{u} = 0$

$\bar{u} = f_1(z) \quad f_2(t)$ (1)

written as

$$f_1(z).\frac{O}{\partial t}[f_2(t)] = Cvf_2(t).\frac{\partial^2}{\partial z^2}[f_1(z)]$$

$$\frac{\frac{\partial^2}{\partial z^2}[f_1(z)]}{f_1(z)} = \frac{\frac{O}{\partial t}[f_1(t)]}{Cvf_2(t)}$$

The left hand term does not contain t and hence is constant if t is considered variable. Similarly the right hand term is constant Z variable constant (say – A^2)

$$\frac{\partial^2}{\partial z^2}[f_1(z) = -A^2 f_1(z) \tag{3}$$

eq : (3) & (4)

$$\frac{\partial^2}{\partial t^2}[\delta_2(t)] = -A^2Cvf_2(t) \tag{4}$$

$$f_1(z) = C_1 CosAz + C_2 SinAz \tag{5}$$

$$f_2(t) = C_3 \in^{-A^2} C_v t \tag{6}$$

C_1, C_2, C_3 = arbitrary constant

$\in =$ base of hyperbolic

Equation (1) becomes

$$\bar{ui} = (C_u CosA_z + CsSinAz) \in^{-A^2} c_v t \tag{7}$$

Equation (7) must

time : t

z = 0 ; $\bar{u} = 0$; $c_u = 0$

equation (7)

$$\bar{u} = C_s(\sin Az) \in^{-A^2} Cvt \tag{8}$$

to ar z = H, $\bar{u} = 0$

$$0 = C_s(SinAH) \in^{-A^2} Cvt \tag{9}$$

equation (9)

AH = n π, where

$$\bar{u} = C_5\left(\frac{Sinn\pi z}{H}\right) \in \frac{-\pi^2}{H^2} Cv.t + B_2\left(\sin\frac{2\pi z}{H}\right) \in \frac{-4\pi^2}{H^2} Cv.t$$

$$+ \ldots\ldots B_n\left(Sin\frac{n\pi z}{H}\right) \in \frac{-n^2\pi^2}{H^2} Cv.t + \ldots\ldots\ldots$$

$$\bar{u} = \sum_{n=1}^{n=\infty} B_n\left(\sin\frac{n\pi z}{H}\right) \in \frac{-n^2\pi^2}{H^2} Cv.t$$

t = 0, $\in \frac{-n^2\pi^2}{H^2} Cv.t = 1$ and $\bar{u} = \overline{u_0}$

$$\therefore \quad \overline{u_0} = \sum_{n=1}^{n=\infty} B_n\left(Sin\frac{n\pi z}{H}\right)$$

$$\int_0^{\pi} \sin nx \quad \sin ax\, dx = 0 \tag{10}$$

$$\int_0^{\pi} Sin^2 nx dx = \frac{\pi}{2} \tag{11}$$

[redacted]

where m and n varied x changed $\frac{\pi z}{H}$, to $\frac{\pi}{H} dz$

$$\int_0^H Sin\frac{m\pi z}{H}\sin\frac{n\pi z}{H}\,dz = 0 \qquad (12)$$

$$\int_0^H Sin^2\frac{n\pi z}{H}dz = \frac{H}{2} \qquad (13)$$

Sides intensity limit 0 to H.

$$\int_0^H \overline{u_0} Sin\frac{n\pi z}{H}\,dz = \sum_{m=1}^{m=\infty} B_m\int_0^H Sin\frac{M\pi z}{H}\,Sin\frac{n\pi z}{H}\,dz + B_n\int_0^H Sin^2\frac{n\pi z}{H}\,dz \qquad (14)$$

Multiplication $Sin\frac{n\pi z}{H}$

Hence $\int_0^H \overline{u_0} Sin\frac{n\pi z}{H}\,dz = B_n\frac{H}{2}$

$$B_n = \frac{2}{H}\int_0^H \overline{u_0} Sin\frac{n\pi z}{H}\,dx \qquad (15)$$

$$\bar{u} = \sum_{n=1}^{n=\infty}\left[\frac{2}{H}\int_0^H \overline{u_0} Sin\frac{n\pi z}{H}\,dz\right]\left[Sin\frac{n\pi z}{H}\right]\frac{e^{-n^2\pi^2}}{H^2}Cvt \qquad (16\quad)$$

$$\bar{u} = \sum_{n=1}^{n=\infty}\frac{2\Delta\sigma}{n\pi}(1 - Cosn\pi)\left(Sin\frac{n\pi z}{H}\right)\in\frac{-n^2\pi^2}{H^2}Cvt \qquad (17)$$

n = even 1 – cos nπ = 0

n = odd 1 – con nπ = 2

$$\bar{u} = \frac{4}{n}\Delta\sigma\sum_{N=1}^{N=\infty}\frac{1}{(2N+1)}\left[\frac{Sin(2N+1)\pi z}{H}\right]\in\frac{(2N+1)^2\pi^2}{H^2}Cv.t$$

[redacted]

[redacted]

$$\Delta_p = mv\Delta\sigma^1 dz$$

$\Delta p^1 =$ effective pressure

$$= \Delta\sigma - \bar{u}$$

$$\Delta P = mv\left(\Delta\sigma - \bar{u}\right)dz$$

$$P = \int_0^H mv\left(\Delta\sigma - \bar{u}\right)dz$$

$$P = mv\Delta\sigma H\left[1 - \frac{8}{\pi^2}\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2} \in \frac{-(2N+1)^2\pi^2}{H^2}Cvt\right]$$

At t = ∞

Pf given by

$$P_f = mv\Delta\sigma H$$

The ratio P to p f,

$$u\% = \frac{P}{p_f}\times 100$$

$$u\% = \left[1 - \frac{8}{\pi^2}\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2} \in \frac{-(2N+1)^2}{H^2}Cv.t\right]\times 100$$

$$Tv = \frac{Cvt}{d^2}$$

$$\left(d = \frac{H}{2}\right)$$

$$u\% = \left[1 - \frac{8}{\pi^2}\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2} \in \frac{-(2N+1)^2}{4} - Tv\right]\times 100$$

$$u\% = f(Tv)$$

[redacted]

The degree of consolidation time function

$$T_v = \frac{K}{m_v \gamma_\omega} \frac{t}{d^2} = \frac{K(1+e_0)}{a_v \gamma_\omega} \frac{t}{d^2}$$

Approximate equation Tv

$$u = 1 - \frac{8}{\pi^2} \left[\sum \frac{-\pi^2}{4} Tv + \frac{1}{9} \sum \in \frac{-9\pi^2}{4} Tv + \frac{1}{2\pi} \in \frac{-25\pi^2}{u} Tvt.... \right]$$

when $u < 60\%; Tv = \frac{\pi}{4} \left(\frac{u}{100} \right)^2$

when $u > 60\%; T_v = -0.09332 \log 10 \left(1 - \frac{u}{100} \right) - 0.0851$

i. Table values of time factor
ii. Double drainage with all inner distribution of consolidation pressure.
iii. Single drainage with uniform distribution of consolidation

u (%)	Tv	u(%)	Tv
5	0.002	55	0.235
10	0.008	60	0.287
15	0.018	65	0.342
20	0.031	70	0.403
25	0.049	75	0.477
30	0.071	80	0.567
35	0.096	85	0.684
40	0.126	90	0.848
50	0.197	100	∞

values of time factor for single drainage

22. Explain the Laboratory Consolidation Test

[redacted]

The Laboratory consolidation test is conducted with an apparatus known as. Consolidometer, consisting essentially of a loading frame and consolidation call in which the specimen in kept. Porous stones are put on the top and bottom ends of the specimen.

Fig. a and fig b show the fixed ring cell and floating ring cell, respectively. In the fixed ring cell, only the top porous stone is permitted to more downwards as the specimen compresses. In the floating ring cell, both top and bottom porous stones are free to compress the specimen towards the middle.

Direct measurement of permeability of the specimen at any stage of loading can be made only in the fixed ring type. However, the floating ring cell has the advantage of having smaller effects of friction between the specimen ring and the soil specimen.

The loading Machine is usually capable of applying speedy vertical pressure, such as 10, 20, 50, 100, 200, 400, 800, 1000 kN/ m^2 (kPa) and each pressure increment is maintained constant until the compression virtually ceases, generally 24 hour. It is measured by means of a dial gauge.

Dial gauge readings are taken after the application of each pressure increment of the following told elapsed times :

0.25,, 1.00,, 2.25, 4.00, 6.25 , 9.00, 12.25, 25, 36, 49, 60 minutes.

The dial gauge readings showing the final compression under each pressure increment are also recorded. After the completion of consolidation under the desired maximum vertical pressure, the specimen is unloaded and allowed to swell. The final dial reading corresponding to the completion of swelling it recorded and the specimen is takes out and dried to determine its water content and the weight of soil solids.

<u>UNIT-IV</u>

SHEAR STRENGTH

[redacted]

Shear strength of cohesive and cohesiveness soils-Mohr, Coulomb failure theory -Saturated soil and unsaturated soil (basic only)-Strength parameter-Measurement of shear strength, direct shear, Tri axial compression, UCC and Vane shear tests-Types of shear stress tests based on drainage and their applicability -Drained and un-drained behavior of clay and sand-Stress path for conventional tri axial test.

Two mark questions and answers

1. What are the tests available for determine the shear strength?

a) Direct shear test

b) Tri axial Shear test

c) Unconfined compression test

d) Vane shear test

1. **What are the advantages for direct shear test?**
2. As test progress, the area under shear gradually decreases. The corrected area at failure should be used in determining the values of σ and τ .
3. As compare to tri axial test, there is little control on the drainage of soil.
4. The plane of shear failure is pre determined which may not be the weakest one.
5. **What are advantages of tri axial tests?**

1) The shear test under all the three drainage conditions can be performed with complete control

2) The precise measurements of the pore pressure and volume change during the test are possible.

3) The stress distribution on the failure plane is uniform

4) The state of stress with in the specimen during any stage of stress, as well as at failure is completely determines.

6. **If angle of internal pressure of a soil is 36^0. Find the angle made by failure plane with respect to minor principle plane.**

The angle made by failure plane with respect to minor principle plane

$$= \frac{90-36}{2}$$

$=27^0$.

4. **C and Φ are not fundamental parameters. But only mathematical parameters of soil. Why?**

Research showed that the parameters C and Φ are not necessity fundamental properties of t he soil as was originally assured by Coulomb. These parameters depend upon a number of factors, such as water content drainage conditions.

The current practice is to consider C and Φ as mathematical parameters which represent the failure conditions for a particular soil under conditions. That is the reason why C and Φ are now called cohesion intersects and angel of shearing resistance.

6. What are pore pressure parameters and write down skempton's pore pressure equation?

Pore pressure parameters express response of pore pressure due change in the total stress under un-drained condition.

Skempton's pore pressure equation

$\Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3))$

Where A and B are the skempton's pore pressure parameters.

7. State coulombs equation for determination of sear strength of soil both for total and effective stress condition.

1. for total stress

Shear strength $\tau_{f} = C + \sigma \tan\phi$

Where, c-cohesion

σ - Total stress

ϕ - Angle of internal friction

2. for effective stress

Shear strength, $\tau_{f} = C' + \sigma' \tan\phi$

Where, σ' = effective stress

8. What are limitations of direct shear test?

1. The stress conditions are known only at failure .the conditions prior to failure are undermine and there, the Mohr circle can not be drawn

2. The stress distribution on the failure is not uniform .the stress are more at the edges and lead to the progressive failure, like tearing of a paper.

3. The area under shear gradually decreases as the test progress. But the corrected area cannot be determined and therefore, the original area is take is for the computation of stress.

4. The orientation of the failure plane is fixed .the plane may not be the weakest plane.

9. Define: normally consolidated soil

A normally consolidated soil is which had not been subjected to a pressure greater than the present exiting pressure.

10. Define: over consolidated soil

A soil is to be over consolidated it had been subjected it had been subjected in the past to a pressure in excess of the present pressure.

11. What are the basic components are constituted in shearing resistance of soil ?

The shearing resistance of soil is constituted basically of the following components:

(1) The structural resistance to displacement of the soil because of the interlocking of the particles,

(2) The frictional resistance to translocation between the individual soil particles at their contact points, and

(3) Cohesion or adhesion between the surfaces of the soil particles.

<u>16 mark questions and answers</u>

1. **Explain the mohr's stress circle**

[redacted]

Through a point in a loaded soil mass, innumerable planes pass and stress components on each plane depends upon the direction of the plane. It can be shown that there exist three typical planes, mutually orthogonal to each other, on which the stress is wholly normal and no shear stress acts.

These planes are called the principal planes and the normal stresses acting on these planes are called the principal stresses. In the order of decreasing magnitude of the normal stress, these planes are called major, intermediate and minor principal planes and the corresponding normal stresses on them are called major principal stress $\sigma 1$, intermediate principal stress $\sigma 2$ and minor principal stress $\sigma 3$. Many problems in soil engineering can be approximated by considering two dimensional stress conditions.

Fig. Shows a soil element subjected to two dimensional stress system. From the consideration of the equilibrium of the element, one gets the following expressions for the normal stress σ and shearing stress τ on any plane MN inclined at an angle α with

The x direction:

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \left(\frac{\sigma_y - \sigma_x}{2} \right) \cos 2\alpha + \tau_{xy} \sin 2\alpha \qquad \text{... (4.1)}$$

And

$$\tau = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha \qquad \text{... (4.2)}$$

Where σy and σx = normal stresses on planes perpendicular to y and x axes, respectively

($\sigma y > \sigma x$) τxy ($=\tau yx$) = shear stresses on these two planes

Squaring Eqs. 4.1 and 4.2 and adding, we get the following results:

$$\left(\sigma - \frac{\sigma_y + \sigma_x}{2} \right) 2 + \tau 2 = \left(\frac{\sigma_y - \sigma_x}{2} \right) 2 + \tau_{xy} 2 \qquad \text{... (4.3)}$$

Eq. 4.3 is the equation of a circle whose centre has co-ordinates

$\left(\frac{\sigma_y + \sigma_x}{2}, 0 \right)$, and whose radius is equal to $\sqrt{\left\{ \frac{1}{2} (\sigma_y - \sigma_x) \right\} 2 + \tau_{xy} 2}$

The co-ordinates of points on the circle represent the normal and shearing stresses on inclined planes at a given point. This circle is known as Mohr's circle of stress (Mohr, 1870).

[redacted]

[redacted]

To draw the Mohr circle, the normal stresses σx and σy are marked on the abscissa, at points B and A and a circle is drawn with point C, mid-way between A and B, as the centre, with radius equal to CB1=CA1 where BB1 and AA1 are the perpendiculars drawn at B and A of magnitude equal to τxy. The sign conventions are shown in Fig.4.1 (b) Fig.4.1 (c) shows the Mohr circle so drawn. Te co-ordinates of any point F (σ, τ) represent the stress conditions on plane which makes an angle α with the x direction.

If from a point B1 [Fig. 4.1 (d)] on a circle representing the state of stress on vertical plane, a line is drawn parallel to this plane (i.e vertical), it intersects the circle at a point P. Also, if from the point A1 on the circle representing the stresses on the horizontal plane, a line is drawn parallel to this latter plane (i.e horizontal) it will also intersect the circle in the same point P. In general, if through a point F representing the stresses on a given plane, a line is drawn parallel to that plane, it will also intersect the circle in the point P. The point P is therefore, a unique point called the origin of planes or the pole.

Let us now take the case of soil element whose sides are the principal planes, i.e consider the state of stress where only normal stresses are acting on the faces of the element. Fig. 4.2 (a) shows the element, and Fig.4.2 (b) shows the Mohr circle.

In Fig. 4.2 (a) the major principal plane is horizontal. Hence the pole P is located by drawing a horizontal line through point A [Fig 4.2 (b)] representing the major principal stress σ1. This intersects the circle at B. If a line PF is drawn through P at an angle α with the horizontal, it will intersect the circle at F which represents the stress conditions on a plane inclined at an angle α with the direction of the major principal plane.

Fig. 4.2 (c) shows an element in which the principal planes are not horizontal and vertical, but are inclined to y and x-directions. Fig. 4.2 (d) shows the corresponding stress circle. Point A represents the major principal stress (σ1, 0) and B represents the minor principal stress (σ3, 0). Hence to get the position of the pole, a line is drawn through a, parallel to the major principal plane, to intersect the circle in P. Evidently, PB gives the direction of minor principal plane. To find the stress components on any plane MN inclined at an angle α with the major principal plane, a line is drawn through P, at an angle α with PA, to intersect the circle at F. The co-ordinates (σ, τ) of point F give the stress components on the plane MN. Analytical expression for σ, τ are :

$$\sigma = \frac{\sigma 1 + \sigma 3}{2} + \frac{\sigma 1 - \sigma 3}{2} \cos 2\alpha \quad \text{... (4.4)}$$

$$\tau = \frac{\sigma 1 + \sigma 3}{2} \sin 2\alpha \quad \text{... (4.5)}$$

The resultant stress on any plane is $\sqrt{\sigma 2 + \tau 2}$ and its angle of obliquity β is equal to tan¹ $\left(\frac{\tau}{\sigma}\right)$

[redacted]

The maximum shear stress (point G) τmax is equal to $\frac{\sigma_1 - \sigma_3}{2}$ and it occurs on planes with $\alpha = 45°$. In Fig. 4.2 (b), PG shows the direction of plane having maximum shear stress. The normal stress on this plane will be equal to $\frac{\sigma_1 + \sigma_3}{2}$

2. Explain the Mohr-coulomb failure theory

1. Material fails essentially by shear. The critical shear stress causing failure depends upon the properties of the material as well as on normal stress on the failure plane.

2. The ultimate strength of the material is determined by the stresses on the potential failure plane (or plane of shear)

3. When the material is subjected to three dimensional principal stress (i.e.$\sigma 1$, $\sigma 2$, $\sigma 3$) the intermediate principal stress does not have any influence on the strength of material. In other words, the failure criterion is independent of the intermediate principal stress.

Note. For detailed discussions on various theories of failure, see Chapter 19, where the effect of the intermediate principal stress has also been discussed.

The theory was first expressed by Coulomb (1776) and later generalized by Mohr. The theory can be expressed algebraically by the equation.

$$\tau_f = s = F(\sigma) \quad \text{... (4.6)}$$

Where τf =s= shear stress on failure plane, at failure = shear resistance of material

F (σ) = function of normal stress

If the normal and shear stress corresponding to failure are plotted, then a curve is obtained. The plot or the curve is called the strength envelope. Coulomb defined the function F (σ) as a linear function of σ and gave the following strength equation:

$$s = c + \sigma \tan \phi \quad \text{... (4.7)}$$

Where, the empirical constants c and ϕ represent respectively, the intercepts on the shear axis, and the slope of the straight line of Eq. 4.7 Fig . These parameters are usually termed as cohesion and angle of internal friction or shearing resistance respectively.

Fig. 4.3 (b) shows the Mohr's envelope, which is the graphical representation of Eq. 4.6. Coulomb considered that the relationship between shear strength and normal stress could be adequately represented by the straight line. The generalized Mohr theory also recognizes that the shear strength depends on the normal stress, but indicates that the relation is not linear. The strength theory upon which the Coulomb and Mohr strength lines are based indicates that definite relationship exists among the principal stresses, the angle of internal friction and the inclination of the failure plane. The curved failure envelope of Mohr is often referred to as a straight line for most of the calculations regarding the stability of soil mass. For an ideal pure friction material, such a straight line passes through the origin [Fig. 4.4 (a)]. However, dense sands exhibit a slightly curved strength line, indicated by dashed line. Fig. 4.4 (b) represents purely cohesive (plastic) material, for which the straight line is parallel to the σ- axis. The strength of such a material is independent of the normal stress acting on the plane of failure. The way in which a straight line is fitted to a Mohr envelope will depend on the range of α which is of interest.

It can, therefore, be concluded that the Mohr envelope can be considered to be straight if the angle of internal friction ϕ is assumed to be constant. Depending upon the properties of a material the failure envelope may be straight or curved, and it may pass through the origin of stress or it may intersect the shear stress axis.

3. Explain the effective stress principle

In Eq. 4.7, it is assumed that the total normal stress governs the shear strength of soil. This assumption is not always correct. Extensive tests on re-mould clays have sustained beyond doubt Terzaghi's early concept that the effective normal stresses control the shearing resistance of soils. Therefore, a failure criterion of greater general applicability is obtained by expressing the shear strength as a function of the effective normal stress

σ', given by the equation : $\tau f = c' + \sigma' \tan \phi'$ .... (4.8)

or $\tau f = c' + (\sigma - u) \tan\phi'$ .... (4.9)

where c' = effective cohesion intercept; andϕ' = effective angle of shearing resistance

In terms of total stresses, the equation takes the form:

$$\tau f = cu + \sigma \tan \phi u \quad .... (4.10)$$

Where cu = apparent cohesion; ϕu = apparent angle of shearing resistance.

The normal stress σ' and shear stress τ on any plane inclined at an angle α to the major principal plane can be expressed in terms of effective major principal stress $\sigma 1'$ and effective minor principal stress $\sigma 3'$ from Eqs. 4.4 And 4.5 as under:

$$\sigma' = \frac{\sigma_{1'} + \sigma_{3'}}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \qquad \text{.... (4.11)}$$

$$\tau = \frac{\sigma_{1'} - \sigma_{3'}}{2} \sin 2\alpha \qquad \text{.... (4.12)}$$

Substituting the values of σ' in Eq. 4.8, we get

$$\tau_f = c' + \tan\phi' \left(\frac{\sigma_{1'} + \sigma_{3'}}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \right) \qquad \text{.... (4.13)}$$

The most dangerous plane i.e, the plane on which failure will take place is the one on which the difference (τf - τ), between the shear strength and shear stress is minimum.

$$(\tau_f - \tau) = c' + \frac{\sigma_1' + \sigma_3'}{2} \tan\phi' + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \tan\phi' - \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha$$

Differentiating this with respect to α, we get

$$\frac{d}{d\alpha}(\tau_f - \tau) = -(\sigma_1' - \sigma_3') \sin 2\alpha \tan\phi' - (\sigma_1' - \sigma_3') \cos 2\alpha$$

For a minimum $(\tau_f - \tau), \frac{d}{d\alpha}(\tau_f - \tau) = 0$

This gives $\cos 2\alpha = -\sin 2\alpha \tan\phi'$ or $\cot 2\alpha = -\tan\phi' = \cot(90° + \phi')$

$$\alpha = \alpha f = 45° + \frac{\phi'}{2} \qquad \text{.... (4.14)}$$

The above expression for the location of the failure plane can be directly derived from the Mohr circle (Fig. 4.5). J F represents the failure envelope given by the straight line $\tau f = c' + \sigma' \tan\phi'$. The pole P will be the point with stress co-ordinates as ($\sigma 3'$, 0). The Mohr circle is tangential to the Mohr envelope at the point F. PF represents the direction of the failure plane, inclined at an angle αf with the direction of the major principal plane. From the geometry of Fig. 18.5, we get from triangle JFK

$$2\alpha f = 90° + \phi' \qquad \text{or} \qquad \alpha f = 45° + \frac{\phi'}{2}$$

It should be noted that for any combination of the applied principle effective stress $\sigma 1'$ and $\sigma 3'$, failure will occur only of the stress circle touches the failure envelope. Also, the coordinates of the failure point F represent the stress componentsσ' and τf at failure. As it is evident from Fig. 4.5, the τf at failure is less than the maximum shear stress, corresponding to the point G, acting on the plane PG. Thus, the failure plane does not carry maximum shear stress, and the plane which has the maximum shear stress is not the failure plane.

4. Explain the direct shear test.

This is a simple and commonly used test and is performed in a shear-box apparatus (Fig. 4.6). The apparatus consists of a two piece shear box of square or circular cross-section. The lower half of the box is rigidly held in position in a container which rests over slides or rollers and which can be pushed forward at a constant rate by geared jack, driven either by electric motor or by hand.

The upper half of the box butts against a proving ring. The soil sample is compacted in the shear box, and is held between metal grids and porous stones (or plates). As shown in Fig. 4.6 (a), the upper half of the specimen is held in the upper box and the lower half in the lower box, and the joint between the two parts of the box is at the level of the centre of the specimen.

Normal load is applied on the specimen from a loading yoke bearing upon steel ball of pressure pad. When a shearing force is applied to the lower box through the feared jack, the movement of lower part of the box is transmitted through the specimen to the upper part of the box and hence on the proving ring.

The deformation of proving ring indicates the shear force. The volume change during the consolidation and during the shearing process is measured by mounting a dial gauge at the top of the box. The soil specimen can be compacted in the shear box by clamping both the parts together with the help of two screws.

These screws are, however, removed before the shearing force is applied. Metal grids, placed above the top and below the bottom of the specimen may be perforated if drained test is required, or plain if un-drained test is required. The metal grids have linear slots or serrations to have proper grip with the soil specimen, and are so oriented that the serrations are perpendicular to the direction of the shearing force.

The specimen of the shear box is sheared under a normal load N. The shearing strain is made to increase at a constant rate, and hence the test is called the strain controlled shear box test. The other type of test is the stress controlled shear box test, in which there is an arrangement to increase the shear stress at a desired rate and measure the shearing strain. Fig. 4.6 (a) shows the strain controlled shear box.

The shear force, F, at failure, corresponding to the normal load N is measured with the help of the proving ring. A number of identical specimens are tested under increasing normal loads and the required maximum shear force is recorded. A graph is plotted between the shear force F as the ordinate and the normal load N as the abscissa. Such a plot gives the failure envelope for the soil under the given test conditions. Fig. 4.6 (c) shows such a failure envelope plotted as a function of the shear stress s and the normal stressσ. The scales of both s and σ are kept equal so that the angle of shearing resistance can be measured directly from the plot.

Any point F (σ, τ) on the failure envelope represents the state of stress in the material during failure, under a given normal stress. In the direct shear test, the failure plane MN is predetermined, and is horizontal. Fig. 4.6 (b) shows the stress conditions during failure. In order to find the direction of principal planes at failure, we first locate the position of the pole on the Mohr circle [Fig 4.6 (c)] on the

principle that the line joining any point on the circle to the pole P gives the direction of the plane on which the stresses are those given by the co-ordinates of that point.

Hence, through point F a horizontal line (representing the direction of the failure plane) is drawn to intersect the circle at the point P which is the pole. Since points A and B represent respectively, the major and minor principal stresses, PA and PB give the directions of major and minor principal planes.

Tests can be performed under all the three conditions of drainage. To conduct un-drained test, plan grids are used. For the drained test, perforated grids are used. The same is first consolidated under the normal load, and then sheared sufficiently slowly so that complete dissipation of pore pressure takes place.

The drained test is therefore also known as the slow test, and the shearing of cohesive soil may sometimes require 2 to 5 days. Cohesion less soils are sheared in relatively less time. For the consolidated un-drained test, perforated grids are used. The sample is permitted to consolidate under the normal load. After the completion of consolidation, the specimen is sheared quickly in about 5 to 10 minutes.

Comments on the shear box test.

The direct shear test is a simple test. The relatively thin thickness of sample permits quick drainage and quick dissipation of pore pressure developed during the test. However, the test has the following disadvantages:

(1) The stress conditions across the soil sample are very complex. The distribution of normal stresses and shearing stresses over the potential surface of sliding is not uniform. The stress is more at the edges and less in the centre. Due to this there is progressive failure of the specimen i.e., the entire strength of the soil is not mobilized simultaneously.

(2) As the test progresses, the area under shear gradually decreases. The corrected area (Af) at failure should be used in determining the values of σ and, τ.

(3) As compared to the tri-axial test, there is little control on the drainage of soil.

(4) The plane of shear failure is predetermined, which may not be the weakest one.

(5) There is effect of lateral restraint by the side walls of the shear box.

5. Explain the tri-axial compression test

The strength test more commonly used in a research laboratory today is the triaxial compression test, first introduced in the U.S.A by A. Casagrande and Karl Terzaghi. The solid specimen, cylindrical in shape, is subjected to direct stresses acting in three mutually perpendicular directions. In the common solid cylindrical specimen test, the major principal stress $\sigma 1$ is applied in the vertical direction, and the other two principal stresses $\sigma 2$ and $\sigma 3$ ($\sigma 2 = \sigma 3$) are applied in the horizontal direction by the fluid pressure round the specimen.

The test equipment specially consists of a high pressure cylindrical cell, made of Perspex or other transparent material, fitted between the base and the top cap. Three outlet connections are generally provided through the base: cell fluid inlet, pore water out let from the bottom of the specimen and the drainage outlet from the top of the specimen.

A separate compressor is used to apply fluid pressure in the cell. Pore pressure developed in the specimen during the test can be measured with the help of a separate pore pressure measuring equipment, such as Bishop's apparatus shown in Fig. 4.8. The cylindrical specimen is enclosed in a rubber membrane. A stainless steel piston running through the centre of the top cap applies the vertical compressive load (called the deviator stress) on the specimen under test.

The load is applied through a proving ring, with the help of a mechanically operated load frame. Depending upon the drainage conditions of the test, solid nonporous discs or end caps, or porous discs are placed on the top and bottom of the specimen and the rubber membrane is sealed on to these end caps by rubber rings.

The length of the specimen is kept about 2 to 2 ½ times its diameter. The cell pressure $\sigma 3$ ($=\sigma 2$) acts all round the specimen; it acts also on the top of the specimen as well as the vertical piston meant for applying the deviator stress. The vertical stress applied by the loading frame, through the proving ring is equal to ($\sigma 1-\sigma 3$), so that the total stress on the top of the specimen = $(\sigma 1 - \sigma 3) + \sigma 3 = \sigma 1$ = major principal stress.

This principal stress difference ($\sigma 1 - \sigma 3$) is called the deviator stress recorded on the proving ring dial. Another dial measures the vertical deformation of the sample during testing. It is desirable to maintain the cell pressure reservoir and mercury control apparatus, devised by Skempton and Bishop (1950), as shown in Fig. 4.9 (a). For long duration test (lasting about a week or more), self-compensating mercury control can be used [Fig. 4.9 (c)].

A particular confining pressure $\sigma 3$ is applied during one observation, giving the value of the other stress $\sigma 1$ at failure. A Mohr circle corresponding to this set of ($\sigma 1$, $\sigma 3$) can thus be plotted. Various sets of observations are taken for different confining pressures $\sigma 3$ and the corresponding values of $\sigma 1$ are obtained. Thus, a number of Mohr circles, corresponding to failure conditions, are obtained. A curve, tangential to these stress circles, gives the failure envelope for the soil under the given drainage conditions of the test.

Shear tests can be performed in the tri-axial apparatus under all the three drainage conditions. For un-drained test, solid (nonporous) end caps are placed on the top and bottom of the specimen. In the consolidated-un-drained test, porous discs are used. The specimen is allowed to consolidate under the desired confining pressure by keeping the pore water outlet open.

When the consolidation is complete, the pore water outlet is closed, and the specimen is sheared under un-drained conditions. The pore water pressure can be measured during the un-drained part of the test. In the drained test, porous discs are used, and the pore water outlet is kept open throughout the test. The compression test is carried out sufficiently slowly to allow for the full drainage during the test.

Measurement of pore pressure during the test.

It mainly consists of (i) the null indicator, (ii) the control cylinder, (iii) pressure gauge, (iv) mercury manometer, and (v) burette.

The null indicator consisting of a single straight section of glass capillary tube dipping into an enclosed trough of mercury, is connected to the tri-axial cell through valve a by a copper tube, and to the contrl cylinder etc. through valve k. An increase in pore pressure in the sample during the test will tend to depress the mercury in the limb of the null indicator. This can be immediately balanced by adjusting the piston in the control cylinder to increase the pressure in the limb by an equal amount which is registered in the pressure gauge. Valves m, f and j are kept closed during the pore pressure measurements. In addition to the pressure gauge, a mercury manometer is also provided. This is used (i) for negative pore pressure, (ii) for accurate measurement of low positive pore pressure, and (iii) for checking the zero error of the pressure gauge.

When this manometer is connected through valves k and m, valves l and n are kept closed. The graduated tube or burette connected to the valve f is used for determining the gauge and manometer readings corresponding to zero pore pressure. In the case of fully saturated samples, this graduated tube can also be used to measure volume change during the consolidation stage of test in which drainage is permitted through the base of specimen (Bishop and Henkel 1957).

6. **Explain the Stress conditions in soil specimen during tri-axial testing.**

Fig. 4.10 (a) shows the effective stresses acting on the soil specimen during tri-axial testing. The minor principal stress and the intermediate principal stress are equal. The effective minor principal stress is equal to the cell pressure minus the pore pressure. The major principal stress is equal to the deviator stress plus the cell pressure.

The effective major principal stress σ_1' is equal to the major principal stress minus the pore pressure. The stress components on the failure plane MN are σ' and τ_f, and the failure plane is inclined at an angle α' to the major principal plane. Fig. 4.10 (b) shows the failure envelope JF and a Mohr circle corresponding to any failure point F. Since $\angle JFC = 90°$ and the failure envelope cuts the abscissa at an angle ϕ', the angle α' of the failure plane is given by :

$$\alpha' = \frac{1}{2}\angle FCA = \frac{1}{2}(90° + \phi') = 45° + \phi'/2 \qquad \text{... (4.14 a)}$$

The principal stress relationship at failure can be found with the help of Fig.4.10 (b)

FC = radius of Mohr circle = $\frac{1}{2}(\sigma_1' - \sigma_3')$; OC = $\frac{1}{2}(\sigma_1' + \sigma_3')$; OK = $c' \cot \phi'$

Hence

$$\sin\phi' = \frac{FC}{KC} = \frac{FC}{KO+OC} = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c'\cot\phi' + \frac{1}{2}(\sigma_1' + \sigma_3')} = \frac{(\sigma_1' - \sigma_3')}{2c'\cot\phi' + (\sigma_1' + \sigma_3')}$$

$\therefore$

$$\sigma_1' - \sigma_3' = 2c'\cot\phi' + (\sigma_1' + \sigma_3')\sin\phi' \qquad \text{... (4.15 a)}$$

or

$$\sigma_1'(1 - \sin\phi') = \sigma_3'(1 + \sin\phi') + 2c'\cot\phi'$$

$\therefore$

$$\sigma_1' = \sigma_3'\frac{(1+\sin\phi')}{(1-\sin\phi')} + 2c'\frac{\cos\phi'}{(1-\sin\phi')} \qquad \text{... (4.15)}$$

or

$$\sigma_1' = \sigma_3'\tan 2\left(45^\circ + \frac{\phi'}{2}\right) + 2c'\tan\left(45^\circ + \frac{\phi'}{2}\right) \qquad \text{... (4.16)}$$

or

$$\sigma_1' = \sigma_3'\tan 2\alpha' + 2c'\tan\alpha' \qquad \text{... (4.17)}$$

or

$$\sigma_1' = \sigma_3' N\phi' + 2c'\sqrt{N\phi'} \qquad \text{... (4.17 a)}$$

where

$$N\phi' = \tan 2\alpha' = \tan 2(45^\circ + \phi'/2)$$

Eq. 4.16 or 4.17 gives principal stress relationship. When the soil is in the state of stress defined by the Eq. 4.16 or 4.17, it is said to be in plastic equilibrium. In terms of total stresses, 4.17 is written as

$$\sigma_{1'} = \sigma_3\tan 2\alpha + 2c_u\tan\alpha \quad \text{.. (4..18) or} \quad \sigma_1 = \sigma_3 N_\phi + 2c_u\sqrt{N_\phi} \quad \text{.. (418a)}$$

where

$$\alpha = 45^\circ + \frac{\phi_u}{2} \, and N_\phi = \tan 2\alpha = \tan 2\left(45^\circ + \frac{\phi_u}{2}\right)$$

In Eq. 4.16, σ_1‘ and σ_3’ are known, and the two unknown are ϕ’ and c’. Hence two sets of observations are required to determine these two unknown parameters. In practice, a number of sets (σ_1’, σ_3’) at failure are observed, and Mohr circles are plotted for each set. A curve drawn tangential to these circles gives the failure envelope [Fig. 4 11 (a)].

[redacted]

Another method of plotting the rest results is in the form of the modified failure envelope which is a function for ½ (σ_1'+σ_3') and ½ (σ_1' - σ_3'). Rewriting Eq.4.15 (a) in the form

½ (σ_1'-σ_3') and d'+ ½ (σ_1' + σ_3') tan ψ'

and comparing it with Eq. 4.5 a, we observe that

Sin ϕ' = tan ψ '……….(18.20 a) and c' = $\dfrac{d'}{\cos\phi'}$ ……(4..20 b)

Eq.4.19 representing the principal stress relationship, is the equation of a straight line having its y-coordinate represented by ½ (σ_1'-σ_3') and x- coordinate represented by ½ (σ_1'+σ_3'). Fig. 4.11(b) shows the modified failure envelop, represented by Eq. 419, in which the slopeψ' and the intercept d' are related to ϕ' and c' through Eq. 4.20.

The line so obtained is often called the K_f line (Lambe,1969). The advantage of this method of plotting the failure envelope is that the averaging of scattered test results is facilitated to a great extent, giving the mean value of the parameters.

The calculation of the deviator stress must be done on the basis of the changed area of cross-section at failure, or during any stage of the relation.

$$A2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$$

Where V_1 – initial volume of the specimen; L_1 = initial length of the specimen

ΔV = change in the volume of the specimen

ΔL = Change in the length of the specimen

The deviator stress σ_d is given by $\sigma_d = \dfrac{Additionalaxiallload}{A2}$;

σ_3 = fluid pressure

1

$\sigma_1 = \sigma_3 + \sigma_d$

[redacted]

Knowing $\sigma 1$, E3 and pore pressure. $\sigma 1'$ and $\sigma 3'$ can be determined.

Advantages of tri-axial test:

(1) The shear test under all the three drainage conditions can be performed with complete control.

(2) Precise measurements of the pore pressure and volume change during the test are possible.

(3) The stress distribution on the failure plane as uniform.

(4) The state of stress within the specimen during any stage of the test, as well as at failure is completely determinate.

7. Explain the un-confined compression test

The unconfined compression test is a special ease of tri-axial compression test in which $\sigma_2 = \sigma_3 = 0$. The cell pressure in the tri-axial cell is also called the confining pressure. Due to the absence of such a confining pressure, the uni-axial test is called the unconfined compression test. The cylindrical specimen of soil is subjected to major principal stress $\sigma 1$ till the specimen fails due to shearing along a critical plane of failure.

In its simplest form, the apparatus consists of a small load frame fitted with a proving ring to measure the vertical stress applied to the soil specimen. Fig4.12. (a) shows an unconfined compression tester (Goyal and Singh, 1958). The deformation of the sample is measured with the help of a separate dial gauge. The ends of the cylindrical specimen are hollowed in the form of cone. The cone seating reduce the tendency of the specimen to become barrel shaped by reducing end-restraints. During the test, load versus deformation readings are taken and a rap is plotted.

When a brittle failure occurs, the proving ring dial indicates a definite maximum load which drops rapidly with the further increase of strain. In the plastic failure, no definite maximum load is indicated. In such a case, the load corresponding to 20% strain is arbitrarily taken as the failure load.

Fig. 4.12. (b), (c) shows the stress conditions, at failure, in the unconfined compression test which is essentially an un-drained test (if it is assumed that no moisture is lost from the specimen during the test). Since $\sigma_3 = 0$, the Mohr circle passes trough the origin which is also the pole.

From Eq. 4.18, we get $\sigma_1\ 2c_u \tan \sigma = 2\ c_u \tan \left[45^\circ \dfrac{\phi u}{2}\right]$ ....(4.22)

In the above equation, there are two unknowns c_u and ϕ_u, which cannot be determined by the unconfined test since a number of tests on the identical specimens give the same value of σ_1. Therefore, the unconfined compression test is generally applicable to saturated clays for which the apparent angle of shearing resistance ϕu is zero. Hence

$$\sigma_1 = 2c_u \qquad \text{.... (4.23)}$$

When the Mohr circle is drawn, its radius is equal to $\sigma_1 / 2 = c_u$. The failure envelope is horizontal. Pf is the failure plane, and the stresses on the failure plane are

$$\sigma = \frac{\sigma_1}{2} = \frac{q_u}{2} \qquad \text{..........(4.24)}$$

and

$$tf = \frac{\sigma_1}{2} = \frac{q_u}{2} = c_u \qquad \text{....(4.25)}$$

Where, q_u = unconfined compressive strength at failure. The compressive stress is calculated on the basis of changed cross-sectional area A2 at failure, which is given by

$$A_2 = \frac{V}{L_1 - \Delta L} = \frac{A_1}{1 - \frac{\Delta L}{L_1}}$$

Where V – initial volume of the specimen;

L_1 = initial length of the specimen

ΔL = Change in length at failure.

8. a. Table, gives observations for normal load and maximum shear force for the specimens of sandy clay tested in the shear box, 36 cm² in area under un-drained conditions. Plot the failure envelope for the soil and determine the value of apparent angles of shearing resistance and the apparent cohesion.

Normal load (N)	**Maximum shear force (N)**
100	110
200	152

[redacted]

300	193
400	235

Solution

Fig. 18.13 shows the plot between the shear force F and the normal load N. From the plot, we

Get $\phi_u = 22°$, and total cohesive force = 70 N.

∴ Unit apparent cohesion $c_u = \dfrac{70}{36} = 1.95$ N/cm²

= **19.5** kN/cm² = **19.5** kPa.

8.b. Samples of compacted, clean dry sand were tested in a shear box, 6 cm x 6 cm and the following results were obtained:

Normal load (N)	**:**	**100**	**200**	**300**	**400**
Peak shear load (N)	**:**	**90**	**181**	**270**	**362**
Ultimate shear load (N)	**:**	**55**	**152**	**277**	**300**

Determine the angle of shearing resistance of the sand in (a) the dense, and (b) the loose state.

Solution.

The value of the shearing resistance of sand, obtained from the peak stress represents the value of ϕ in its initial compacted state, while that obtained from the ultimate shear corresponds to the sand when loosened by the shearing action.

[redacted]

Fig.4.14 shows the two plots. The values of angles of shearing resistance are found to be :

(a) dense state : $\phi = 42°$
(b) loose state : $\phi = 37°$

9. A cylindrical specimen of saturated clay, 4 cm in diameter and 9 cm in over all length is tested in an unconfined compression tester. The specimen has coned ends and its length between the apices of cones is 8 cm. Find the unconfined compressive strength of clay, if the specimen fails under an axial load of 46.5 N. The change in the length of specimen at failure is 1 cm.

Solution.

Original length of specimen = 9 cm overall, and 8 cm to apices of cones. Length of cylinder of the same volume and diameter (average length) L1 = 8.66 cm.

Initial cross-sectional area $$A_1 = \frac{\pi \times 4^2}{4} = 12.57 cm^2$$

Change in length at failure, $\Delta L = 1cm$

Area of failure $$= A2 = \frac{A_1}{1 - \frac{\Delta L}{L_1}} = \frac{12.57}{1 - \frac{1}{8.66}} = 14.2 cm^2$$

$\therefore$ Unconfined compressive strength $$q_u - \frac{failureload}{A_2} = \frac{465}{14.2} = 32.8 kN / cm^2$$

= 328 kN/m^2 = 328 k Pa

$\therefore$ Shear strength $$c_u = \frac{q^u}{2} = \frac{328}{2} 165 kN / m^2 = 165 kPa$$

[redacted]

10. A cylinder of soil fails under an axial vertical stress of 160 kN/m², when it is laterally unconfined. The failure plane makes an angle of 50° with the horizontal. Calculate the value of cohesion and the angle of internal friction of the soil.

Solution.

$$\alpha = 50^\circ = 45^\circ; \quad \therefore \phi u = 2(50^\circ - 45^\circ) = 10^\circ$$

$$\tan\ \sigma = \tan \left[45^\circ + \frac{\phi u}{2}\right] = \tan 50^\circ = 1.192$$

As the sample is un-confined, $\sigma_3 = 0$

Now $\sigma_1 = \sigma_3 \tan^2\sigma + 2\ c \tan \sigma$

$160 = 2c \tan 50° = 2c \times 1.192$

$$c = \frac{160}{2 \times 1.192} = 67.1 kN/m^2 = 67.1 kPa.$$

11. Two identical specimens, 4 cm in diameter and 8 cm high, of partly saturated compacted soil is tested in a triaxial cell under un-drained conditions. The first specimen failed at an additional axial load (i.e. deviator load) of 720 N under a cell pressure of 100 kN/m². The second specimen failed at an additional axial load of 915 N under a cell pressure of 200 kN/m². The increase in volume of the first specimen at failure is 1.2 ml and it shortens by 0.6 cm, at failure. The increase in volume of the second specimen at failure is 1.6 ml, and it shortens by 0.8 cm at failure. Determine the value if apparent cohesion and the angle of shearing resistance (a) analytically, (b) graphically by Mohr's circle.

Solution : (a) For the first specimen :

Initial area $$A1 = \frac{\pi \times 4^2}{4} = 12.57 cm^2$$

[redacted]

Initial Volume $V1 = 12.57 \times 8 = 100.56 \text{ cm}^3$

$\therefore$ Area of failure = $$A_2 = \frac{V_1 + \Delta V}{L_1 - \Delta L} = \frac{100.56 + 1.2}{8 - 0.6} = 13.75 cm^2$$

(b) For the second Specimen.

$\therefore$ Deviator stress at failure $$\sigma_d = \frac{720}{123.75} = 52.4 N/cm^2 = 524 kN/m^2$$

$\sigma_3 = 100 \text{ k N / m }^2$; $\sigma_1 = \sigma_3 + \sigma_d = 100 + 524 = 624 \text{ k N /m}^2$

Substituting the value of σ1 and σ3 in Eq 4.18 a,we get

$$624 = 100 N_\phi + 2cu\sqrt{N}_\phi$$

(b) For the second specimen

$A_1 = 12.57 cm^2$; $V_1 = 100.56 cm^2$; $\Delta L = 0.8$; $\Delta V = +1.6 \text{ cm}^3$

$$A_2 = \frac{100.56 + 1.6}{8 - 0.8} = 14.2 cm^2$$ and $$\sigma_d = \frac{915}{14.2} = 64.4 N/cm^2 = 644 kN/m^2$$

$\sigma_3 = 200 \text{ k N / m}^2$; $\therefore \sigma_1 = \sigma_3 + \sigma_d = 200 + 644 = 844 \text{ k N / m}^2$

Substituting the value of σ1 and σ3 in Eq. 4.18 a, we get

$$844 = 200 n_\phi + 2CU\sqrt{n_\phi}$$

Solving (1) and (2), we get ..... (2)

$C_u = 13.6 \text{ k N / m}^3$ (136 k Pa)

And $N\phi = 2.2$

Now $$N = \tan^2\left[42^\circ + \frac{\phi u}{2}\right] = 2.2$$

∴ ϕu = 22°

12. A saturated specimen of cohesion-less sand was tested in triaxial compression and the sample failed at a deviator stress of 482 kN/m² when the cell pressure was 100 kN/m², under the drained conditions. Find the effective angle of shearing resistance of sand. What would be the deviator stress and the major principal stress at failure for another identical specimen of sand, if it is tested under cell pressure of 200 kN/m²?

Solution.

In the drained tests, the effective stresses are equal to the total stress.

$$\sigma_3' = \sigma_3 = 100 kN/m^2 (kPa)$$

$$\sigma_1' = \sigma_3 = \sigma_d = 100 + 482 = 582 m^2 (kPa)$$

Fig. shows the Mohr circle (circle 1). The failure envelope will pass through the origin, since c' =0 for sand, and will be tangential to the circle. The angle of inclination of the failure envelope giveϕ' = 45°.

Alternatively, from Eq4.16

$$\sigma_1' = \sigma_3' \tan^2\left[45^\circ + \frac{\phi'}{2}\right]$$

∴ $$582 = 100 \times \tan 2\left[45 + \frac{\phi'}{2}\right]$$

∴ $$45^\circ + \frac{\phi'}{2} = \tan^{-1}(\sqrt{5.82}) = 67.5^\circ$$

[redacted]

∴ $\phi' = 45°$

For the second specimen with $\sigma_3' = 200$ kN/m², the centre of the Mohr circle passes through $\sigma_3' = 200$ kN/m², and is tangential to the failure envelope. Circle II corresponds to this, from which $\sigma_d' =$ 1160 kN/m²

$\sigma_2 = \sigma_{1'} - \sigma_3' = 960$ kN/m² (960 kPa)

Alternatively, σ_1' can be calculated from the relation :

$$\sigma_1' = \sigma_3' \tan^2\left[42° + \frac{\phi'}{2}\right] = 200 \tan^2\left[42° + \frac{45°}{2}\right] = 1164 kN/m^2 \,(1164 kPa)$$

$\sigma_d = 1164 - 200 = 964$ kN/m² (964 kPa).

13. Following are the results of un-drained tri-axial compression test on two identical soil specimens, at failure:

Lateral pressure σ_3 (kN/m²)	**100**	**300**
Total vertical pressure σ_1(kN/m²)	**440**	**760**
Pore water pressure u (kN/m²)	**- 20**	**60**

Determine the cohesion and angle of shearing resistance (a) referred to total stress, (b) referred to effective stress.

Solution: the circles A and B with dark lines correspond to the total stress, and from failure envelope, drawn tangential to the two circles, we get

$\phi_u = 14°$ and $c_u = 110$ kN /m² (kPa)

For the effective stress analysis, we have

σ_1' : 440 + 20 = 460 and

760 – 60 = 700 kN /m²

σ_3' : 100 + 20 = 120 and

300 – 60 = 240 kN /m²

[redacted]

The Mohr circles A' and B' corresponding to the effective stresses are shown by dotted lines. From the dotted failure envelope, we get

$$\phi' = 20°\ ,\ c' = 76\ \text{kN}/\text{m}^2\ (\text{kPa})$$

14. Un-drained triaxial tests are carried out on four identical specimens of silt clay, and the following results are obtained:

Cell pressure (kN/m²)	**50**	**100**	**150**	**200**
Deviator stress at failure (kN/m²)	**350**	**440**	**530**	**610**
Pore pressure (kN/m²)	**5**	**10**	**12**	**18**

Determine the value of the effective angles opf shearing resistance and the cohesion intercept by plotting (a) conventional failure envelope from Mohr circles, (b) modified failure envelope.

Solution : Table shows the necessary calculations of plotting the failure envelope :

TABLE

Specimen No.	σ_3	σ_3'	σ_d	σ_1'	$\frac{1}{2}(\sigma_1'+\sigma_3')$	$\frac{1}{2}(\sigma_1'-\sigma_3')$
1	50	45	350	395	220	175
2	100	90	440	530	310	220
3	150	138	530	668	403	265
4	200	182	610	792	487	305

Fig. 4.20 shows the conventional failure envelope from Mohr's circles, from which we get ϕ' =29.5° and c' = 8 kN/m² (kPa).

Fig.4.21 shows the modified failure envelope, from which we get ψ'=26.5° and d'=70°

$$\therefore \qquad \sin\phi' = \tan\psi' = \tan 26.5° \qquad \text{or} \qquad \phi' = 30°$$

[redacted]

$$c' = \frac{d'}{\cos\phi'} = \frac{70}{0.86} = 81kN/m^2\ (kPa)$$

15. Explain the vane shear test

Vane shear test is a quick test, used either in the laboratory or in the field, to determine the undrained shear strength of cohesive soil. The vane shear tester consists of four thin steel plates, called vanes, welded orthogonally to a steel rod. A torque measuring arrangement, such as a calibrated torsion spring, is attached to the rod which is rotated by a worm gear and worm wheel arrangement. After pushing the vanes gently into the soil, the torque rod is rotated at a uniform speed (usually at 1° per minute).

The rotation of the vane shears the soil along a cylindrical surface. The rotation of the spring in degrees is indicated by a pointer moving on a graduated dial attached to the worm wheel shaft. The torque T is then calculated by multiplying the dial reading with the spring constant. A typical laboratory vane is 20 mm high and 12 mm in diameter with blade thickness from 0.5 to 1 mm, the blades being made of high tensile steel. The field shear vane is from 10 to 20 cm in height and from 5 to 10 cm in diameter, with blade thickness of about 2.5 mm.

Let τf = unit strength of the soil

H = height of the vane

D = diameter of the vane

Let us assume that the top end of the vane is embedded in the soil so that both top and bottom ends partake in the shearing of the soil. Assuming that the shear resistance of the soil is developed uniformly on the cylindrical surface, the maximum total shear resistance, at failure, developed along the cylindrical surface

$$= \pi\, d\, H\, \tau_f \qquad \text{..... (i)}$$

To find the maximum shear resistance developed at top and bottom ends, consider a radius r of the sheared surface. The shear strength of a right of thickness *dr* will be $2\pi r\, dr\, \tau_f$. Hence the total resistance of both top and bottom faces will be

[redacted]

[redacted]

$$= 2\int_{0}^{\pi} (2\pi r dr)\tau_f \qquad \text{..... (ii)}$$

To total shear strength developed will be equal to the sum of (i) and (ii). The maximum moment of the total shear resistance about the axis of torque rod equals the torque T at failure. Hence

$$T = (\tau d H \tau_f \frac{d}{2} + 2\int_{0}^{d/2} (2\pi r dr \tau_f) r = \pi \tau_f \left[\frac{d^2 H}{2} + \frac{d^3}{6}\right] = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right] \qquad \text{... (4.27)}$$

If only the bottom end partakes in the shearing the above equation takes the from:

$$T = \tau d^2 \tau_f \left[\frac{H}{2} + \frac{d}{12}\right] \qquad \text{.... (4.28)}$$

Knowing T, H and d, the shear strength τ_f can be determined.

[redacted]